Mathematical Programming in Practice

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In management science, as in most sciences, there is a natural interplay between theory andpractice. Theory provides tools for applied work and suggests viable approaches to problem solving, whereas practice adds focus to the theory by suggesting areas for theoretical development in its continual quest for problem-solving capabilities. It is impossible, then, to understand fully either theory or practice in isolation. Rather, they need to be considered as *continually interacting* with one another.

Having established linear programming as a foundation for mathematical programming theory, we now are in a position to appreciate certain aspects of implementing mathematical programming models. In the next three chapters, we address several issues of mathematical-programming applications, starting with a general discussion and followed by specific applications. By necessity, at this point, the emphasis is on linear programming; however, most of our comments apply equally to other areas of mathematical programming, and we hope they will provide a useful perspective from which to view forthcoming developments in the text.

This chapter begins by presenting two frameworks that characterize the decision-making process. These frameworks suggest how modeling approaches to problem solving have evolved; specify those types of problems for which mathematical programming has had most impact; and indicate how other techniques can be integrated with mathematical-programming models. The remainder of the chapter concentrates on mathematical programming itself in terms of problem formulation and implementation, including the role of the computer. Finally, an example is presented to illustrate much of the material.

5.1 THE DECISION-MAKING PROCESS

Since management science basically aims to improve the quality of decision-making by providing managers with a better understanding of the consequences of their decisions, it is important to spend some time reflecting upon the nature of the decision-making process and evaluating the role that quantitative methods, especially mathematical programming, can play in increasing managerial effectiveness.

There are several ways to categorize the decisions faced by managers. We would like to discuss two frameworks, in particular, since they have proved to be extremely helpful in generating better insights into the decision-making process, and in defining the characteristics that a sound decision-support system should possess. Moreover, each framework leads to new perceptions concerning model formulation and the role of mathematical programming.

Anthony's Framework: Strategic, Tactical, and Operational Decisions

The first of these frameworks was proposed by Robert N. Anthony.* He classified decisions in three categories: strategic planning, tactical planning, and operations control. Let us briefly comment on the characteristics of each of these categories and review their implications for a model-based approach to support management decisions.

The examples given to illustrate specific decisions belonging to each category are based primarily on the production and distribution activities of a manufacturing firm. This is done simply for consistency and convenience; the suggested framework is certainly appropriate for dealing with broader kinds of decisions.

a) *Strategic Planning*

Strategic planning is concerned mainly with establishing managerial policies and with developing the necessary resources the enterprise needs to satisfy its external requirements in a manner consistent with its specific goals. Examples of strategic decisions are major capital investments in new production capacity and expansions of existing capacity, merger and divestiture decisions, determination of location and size of new plants and distribution facilities, development and introduction of new products, and issuing of bonds and stocks to secure financial resources.

These decisions are extremely important because, to a great extent, they are responsible for maintaining the competitive capabilities of the firm, determining its rate of growth, and eventually defining its success or failure. An essential characteristic of these strategic decisions is that they have long-lasting effects, thus mandating long planning horizons in their analysis. This, in turn, requires the consideration of uncertainties and risk attitudes in the decision-making process. Moreover, strategic decisions are resolved at fairly high managerial levels, and are affected by information that is both external and internal to the firm. Thus, any form of rational analysis of these decisions necessarily has a very broad scope, requiring information to be processed in a very aggregate form so that all the dimensions of the problem can be included and so that top managers are not distracted by unnecessary operational details.

b) *Tactical Planning*

Once the physical facilities have been decided upon, the basic problem to be resolved is the effective allocation of resources (e.g., production, storage, and distribution capacities; work-force availabilities; marketing, financial, and managerial resources) to satisfy demand and technologicalrequirements, taking into account the costs and revenues associated with the operation of the resources available to the firm. When we are dealing with several plants, many distribution centers, and regional and local warehouses, having products that require complex multistage fabrication and assembly processes, and serving broad market areas affected by strong randomness and seasonalities in their demand patterns, these decisions are far from simple. They usually involve the consideration of a medium-range time horizon divided into several periods, and require significant aggregation of the information to be processed. Typical decisions to be made within this context are utilization of regular and overtime work force, allocation of aggregate capacity resources to product families, definition of distribution channels, selection of transportation and transshipment alternatives, and allocation of advertising and promotional budgets.

c) *Operations Control*

After making an aggregate allocation of the resources of the firm, it is necessary to deal with the day-to-day operational and scheduling decisions. This requires the complete *dis*aggregation of the information generated at higher levels into the details consistent with the managerial procedures followed in daily activities. Typical decisions at this level are the assignment of customer orders to individual machines, the sequencing of these

* Robert N. Anthony, *Planning and Control Systems: A Framework for Analysis*, Harvard University Graduate School of Business Administration, Boston, 1965.

orders in the work shop, inventory accounting and inventory control activities, dispatching, expediting and processing of orders, vehicular scheduling, and credit granting to individual customers.

These three types of decisions differ markedly in various dimensions. The nature of these differences, expressed in relative terms, can be characterized as in Table 5.1.

Table 5.1 Distinct Characteristics of Strategic, Tactical, and Operational Decisions

Characteristics	*Strategic planning*	*Tactical planning*	*Operations control*
Objective	Resource acquisition	Resource utilization	Execution
Time horizon	Long	Middle	Short
Level of management involvement	Top	Medium	Low
Scope	Broad	Medium	Narrow
Source of information	(External	& Internal)	Internal
Level of detail of information	Highly aggregate	Moderately aggregate	Low
Degree of uncertainty	High	Moderate	Low
Degree of risk	High	Moderate	Low

Implications of Anthony's Framework: A Hierarchical Integrative Approach

There are significant conclusions that can be drawn from Anthony's classification, regarding the nature of the model-based decision-support systems. First, strategic, tactical, and operational decisions cannot be madein isolation because they interact strongly with one another. Therefore, an integrated approach is required in order to avoid suboptimization. Second, this approach, although essential, cannot be made without decomposing the elements of the problem in some way, within the context of a hierarchical system that links higher-level decisions with lower-level ones in an effective manner. Decisions that are made at higher levels of the system provide constraints for lower-level decision making, and decision-makers must recognize their impact upon lower-level operations.

This hierarchical approach recognizes the distinct characteristics of the type of management participation, the scope of the decision, the level of aggregation of the required information, and the time frame in which the decision is to be made. In our opinion, it would be a serious mistake to attempt to deal with all these decisions at once via a monolithic system (or model). Even if computer and methodological capabilities would permit the solution of large, detailed integrated models—clearly not the case today—that approach is inappropriate because it is not responsive to management needs at each level of the organization, and would prevent interactions between models and managers at each organization echelon.

In designing a system to support management decision-making, it is imperative, therefore, to identify ways in which the decision process can be partitioned, to select adequate models to deal with the individual decisions at each hierarchical level, to design linking mechanisms for the transferring of the higher-level results to the lower hierarchical levels, which include means to disaggregate information, and to provide quantitative measures to evaluate the resulting deviations from optimal performance at each level.

Mathematical programming is suited particularly well for supporting tactical decisions. This category of decisions, dealing with allocation of resources through a middle-range time horizon, lends itself quite naturally to representation by means of mathematical-programming models. Typically, tactical decisions generate models with a large number of variables and constraints due to the complex interactions among the choices available to the decision-maker. Since these choices are hard to evaluate on merely intuitive grounds, a decision-maker could benefit greatly from a model-based support effort. Historically, mathematical programming has been the type of model used most widely in this capacity, and has contributed a great deal to improving the quality of decision-making at the tactical level.

As we indicated before, tactical decisions are affected by only moderate uncertainties. This characteristic is useful for the application of linear programming, since models of this kind do not handle uncertainties directly. The impact of moderate uncertainties, however, can be assessed indirectly by performing sensitivity analysis. Furthermore, sensitivity tests and shadow price information allow the decision-maker to evaluate how well the resources of the firm are balanced.

For example, a tactical linear-programming model designed to support production-planning decisions might reveal insufficient capacity in a given stage of the production process. The shadow price associated with that capacity constraint provides a local indication of the payoff to be obtained from a unit increase in that limited capacity.

The role of mathematical programming in supporting strategic and operational decisions has been more limited. The importance of uncertainties and risk in strategic decisions, and the great amount of detailed information necessary to resolve operational problems work against the direct use of mathematical programming at these levels. In the decision-making hierarchy, mathematical-programming models become the links between strategic and operational decisions. By carefully designing a sequence of model runs, the decision-maker can identify bottlenecks or idle facilities; and this information provides useful feedback for strategic decisions dealing with acquisition or divestment of resources. On the other hand, by contributing to the optimalallocation of the aggregate resources of the firm, mathematical-programming models generate the broad guidelines for detailed implementation. Operational decisions are reduced, therefore, to producing the appropriate disaggregation of the tactical plans suggested by the mathematical-programming model against the day-to-day requirements of the firm.

The design of a hierarchical system to support the overall managerial process is an art that demands a great deal of pragmatism and experience. In Chapter 6 we describe anintegrated system to deal with strategic and tactical decisions in the aluminum industry. The heart of the system is formed by two linear-programming models that actively interact with one another. In Chapter 10 we analyze a hierarchical system to decideon the design of a job-shop facility. In that case a mixed-integer programming model and a simulation model represent tactical and operational decisions, respectively. Chapter 7 presentsanother practical application of linear programming, stressing the role of sensitivity analysis in coping with future uncertainties. Chapter 14 discusses the use of decomposition for bond-portfolio decisions.

Simon's Framework: Programmed and Nonprogrammed Decisions

A second decision framework that is useful in analyzing the role of models in managerial decision-making was proposed by Herbert A. Simon.* Simon distinguishes two types of decisions: *programmed* and *non-programmed*, which also are referred to as *structured* and *unstructured* decisions, respectively. Although decisions cover a continuous spectrum between these two extremes, it is useful first to analyze the characteristics of these two kinds.

a) *Programmed Decisions*

Programmed decisions are those that occur routinely and repetitively. They can be structured into specific procedural instructions, so they can be delegated without undue supervision to the lower echelons of the organization. As Simon put it, programmed decisions are normally the domain of clerks. This does not necessarily imply that high-level managers do not have their share of programmed decision-making; it simply indicates that the incidence of programmed decisions increases the lower we go in the hierarchy of the organization.

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* Herbert A. Simon, *The Shape of Automation for Men and Management*, Harper and Row, 1965.

b) *Nonprogrammed Decisions*

Nonprogrammed decisions are complex, unique, and unstructured. They usually do not lend themselves to a well defined treatment, but rather require a large amount of good judgment and creativity in order for them to be handled effectively. Normally, top managers are responsible for the more significant nonprogrammed decisions of any organization.

Implications of Simon's Framework: The Degree ofAutomation of the Decision-Making Process

Simon's framework is also very helpful in identifying the role of models in the decision-making process. Its implications are summarized in Table 5.2 which shows the contribution of both conventional and modern methods to support programmed and nonprogrammed decisions.

Table 5.2 The Implications of Simon's Taxonomy

Type of decision	*Conventional methods*	*Modern methods*
Programmed (Structured) Routine Repetitive	Organizational Structure Procedures and regulations Habit-forming	Electronic data processing Mathematical models
Nonprogrammed (Unstructured) Unique Complex	Policies Judgment and intuition Rules of thumb Training and promotion	Hierarchical system design Decision theory Heuristic problem solving

A major issue regarding programmed decisions is how to develop a systematic approach to cope with routine situations that an organization faces on a repetitive basis. A traditional approach is the preparation of written procedures and regulations that establish what to do under normal operating conditions, and that signal higher-management intervention whenever deviations from current practices occur. If properly designed and implemented, these procedures tend to create desirable patterns of behavior and habits among the personnel dealing with routine decisions. Control mechanisms normally are designed to motivate people, to measure the effectiveness of the decisions, and to take corrective actions if necessary.

What allows the proper execution and management of programmed decisions is the organizational structure of the firm, which breaks the management functions into problems of smaller and smaller scope. At the lower echelons of the organization, most of the work assumes highly structured characteristics and, therefore, can be delegated easily to relatively unskilled personnel.

During the last twenty years, we have witnessed a tremendous change in the way programmed decisions are made. First, the introduction of computers has created new capabilities to store, retrieve, and process huge amounts of information. When these capabilities are used intelligently, significant improvements can be made in the quality of decision-making at all levels. Second, the data bank that usually is developed in the preliminary stages of computer utilization invites the introduction of models to support management decisions. In many situations, these models have been responsible for adding to the structure of a given decision. Inventory control, production planning, and capital budgeting are just a few examples of managerial functions that were thought of as highly nonprogrammed, but now have become significantly structured.

Conventional methods for dealing with nonprogrammed decisions rely quite heavily on the use of judgment. Policies and rules of thumb sometimes can be formulated to guide the application of sound judgment and lessen the chances that poor judgment is exercised. Since good judgment seems to be the most precious element for securing high-quality decision-making in nonprogrammed decisions, part of the conventional efforts are oriented toward the recognition of those individuals who possess this quality. Management-development programs attempt to stimulate the growth of qualified personnel, and promotions tend to raise the level of responsibility of those who excel in their managerial duties.

One of the greatest disappointments for the advocates of management science is that, in its present form, it has not had a strong impact in dealing with nonprogrammed decisions. Broad corporate models have been constructed to help the coordination of the high-level managerial functions. Also, hierarchical systems are beginning to offer meaningful ways to approach problems of the firm in which various echelons of managers are involved. In addition, disciplines like decision theory and heuristic problem-solving have made contributions to the effective handling of nonprogrammed decisions. However, it is fair to say that we are quite short of achieving all the potentials of management science in this area of decision-making. This situation is changing somewhat with all the interest in unstructured social problems such as energy systems, population control, environmental issues, and so forth.

We can conclude, therefore, that the more structured the decision, the more it is likely that a meaningful model can be developed to support that decision.

5.2 STAGES OF FORMULATION, SOLUTION, AND IMPLEMENTATION

Having seen where mathematical programming might be most useful and indicated its interplay with other managerial tools, we will now describe an orderly sequence of steps that can be followed for a systematic formulation, solution, and implementation of a mathematical-programming model. These steps could be applied to the development of any management-science model. However, due to the nature of this book, we will limit our discussions to the treatment of mathematical-programming models.

Although the practical applications of mathematical programming cover a broad range of problems, it is possible to distinguish five general stages that the solution of any mathematical-programming problem should follow.

A. Formulating the model.
B. Gathering the data.
C. Obtaining an optimal solution.
D. Applying sensitivity analysis.
E. Testing and implementing the solution.

Obviously, these stages are not defined very clearly, and they normally overlap and interact with each other. Nevertheless we can analyze, in general terms, the main characteristics of each stage.

A) Formulating the Model

The first step to be taken in a practical application is the development of the model. The following are elements that define the model structure:

a) *Selection of a Time Horizon*

One of the first decisions the model designer has to make, when applying mathematical programming to a planning situation, is the selection of the time horizon (also referred to as planning horizon, or cutoff date). The time horizon indicates how long we have to look into the future to account for all the significant factors of the decision under study. Its magnitude reflects the future impact of the decision under consideration. It might cover ten to twenty years in a major capital-investment decision, one year in an aggregate production-planning problem, or just a few weeks in a detailed scheduling issue.

Sometimes it is necessary to divide the time horizon into several time periods. This is done in order to identify the dynamic changes that take place throughout the time horizon. For example, in a production-planning problem it is customary to divide the one-year time horizon into time periods of one month's duration to reflect fluctuations in the demand pattern due to product seasonalities. However, if the model is going to be updated and resolved at the beginning of each month with a rolling one-year time horizon, it could be advantageous to consider unequal time periods; for example, to consider time periods of one month's duration during the first three months, and then extend the duration of the time periods to three months, up to the end of the time horizon. This generally will not adversely affect the quality of the decisions and will reduce the computational burden on the model significantly.

b) *Selection of Decision Variables and Parameters*

The next step in formulating the mathematical-programming model is to identify the decision variables, which are those factors under the control of the decision-maker, and the parameters, which are beyond the control of the decision-maker and are imposed by the external environment.

The decision variables are the answers the decision-maker is seeking. If we are dealing with production-planning models, some relevant decision variables might be the amount to be manufactured of each product at each time period, the amount of inventory to accumulate at each time period, and the allotment of man-hours of regular and overtime labor at each time period.

On some occasions, a great amount of ingenuity is required to select those decision variables that most adequately describe the problem being examined. In some instances it is possible to decrease the number of constraints drastically or to transform an apparent nonlinear problem into a linear one, by merely defining the decision variables to be used in the model formulation in a different way.

The parameters represent those factors which affect the decision but are not controllable directly (such as prices, costs, demand, and so forth). In deterministic mathematical-programming models, all the parameters are assumed to take fixed, known values, where estimates are provided via point forecasts. The impact of this assumption can be tested by means of sensitivity analysis. Examples of some of the parameters associated with a production-planning problem are: product demands, finished product prices and costs, productivity of the manufacturing process, and manpower availability.

The distinction made between parameters and decision variables is somewhat arbitrary and one could argue that, for a certain price, most parameters can be controlled to some degree by the decision-maker. For instance, the demand for products can be altered by advertising and promotional campaigns; costs and prices can be increased or decreased within certain margins, and so on. We always can start, however, from a reference point that defines the appropriate values for the parameters, and insert as additional decision variables those actions the decision-maker can make (like promotions or advertising expenditures) to create changes in the initial values of the parameters. Also, shadow-price information can be helpful in assessing the consequences of changing the values of the initial parameters used in the model.

c) *Definition of the Constraints*

The constraint set reflects relationships among decision variables and parameters that are imposed by the characteristics of the problem under study (e.g., the nature of the production process, the resources available to the firm, and financial, marketing, economical, political, and institutional considerations). These relationships should be expressed in a precise, quantitative way. The nature of the constraints will, to a great extent, determine the computational difficulty of solving the model.

It is quite common, in the initial representation of the problem, to overlook some vital constraints or to introduce some errors into the model description, which will lead to unacceptable solutions. However, the mathematical programming solution of the ill-defined model provides enough information to assist in the detection of these errors and their prompt correction. The problem has to be reformulated and a new cycle has to be initiated.

d) *Selection of the Objective Function*

Once the decision variables are established, it is possible to determine the objective function to be minimized or maximized, provided that a measure of performance (or effectiveness) has been established and can be associated with the values that the decision variables can assume. This measure of performance provides a selection criterion for evaluating the various courses of action that are available in the situation being investigated. The most common index of performance selected in business applications is *dollar value*; thus, we define the objective function as the minimization of cost or the maximization of profit. However, other objective functions could become more relevant in some instances. Examples of alternative objectives are:

Maximize total production, in units.
Minimize production time.
Maximize share of market for all or some products.
Maximize total sales, in dollars or units.
Minimize changes of production pattern.
Minimize the use of a particular scarce (or expensive) commodity.

The definition of an acceptable objective function might constitute a serious problem in some situations, especially when social and political problems are involved. In addition, there could be conflicting objectives, each important in its own right, that the decision-maker wants to fulfill. In these situations it is usually helpful to define multiple objective functions and to solve the problem with respect to each one of them separately, observing the values that all the objective functions assume in each solution. If no one of these solutions appears to be acceptable, we could introduce as additional constraints the minimum desirable performance level of each of the objective functions we are willing to accept, and solve the problem again, having as an objective the most relevant of those objective functions being considered. Sequential tests and sensitivity analysis could be quite valuable in obtaining satisfactory answers in this context.

Another approach available to deal with the problem of multiple objectives is to unify all the conflicting criteria into a single objective function. This can be accomplished by attaching weights to the various measures of performance established by the decision-maker, or by directly assessing a multiattribute preference (or utility function) of the decision-maker. This approach, which is conceptually extremely attractive but requires a great deal of work in implementation, is the concern of a discipline called *decision theory* (or decision analysis) and is outside the scope of our present work.*

B) Gathering the Data

Having defined the model, we must collect the data required to define the parameters of the problem. The data involves the objective-function coefficients, the constraint coefficients (also called the matrix coefficients) and the righthand side of the mathematical-programming model. This stage usually represents one of the most time-consuming and costly efforts required by the mathematical-programming approach.

C) Obtaining an Optimal Solution

Because of the lengthy calculations required to obtain the optimal solution of a mathematical-programming model, a digital computer is invariably used in this stage of model implementation. Today, all the computer manufacturers offer highly efficient codes to solve linear-programming models. These codes presently can handle general linear-programming problems of up to 4000 rows, with hundreds of thousands of decision variables, and are equipped with sophisticated features that permit great flexibility in their operation and make them extraordinarily accurate and effective. Instructions for use of these codes are provided by the manufacturers; they vary slightly from one computer firm to another.

Recently, very efficient specialized codes also have become available for solving mixed-integer programming problems, separable programming problems, and large-scale problems with specific structures (using generalized upper-bounding techniques, network models, and partitioning techniques). Models of this nature and their solution techniques will be dealt with in subsequent chapters.

When dealing with large models, it is useful, in utilizing computer programs, to input the required data automatically. These programs, often called matrix generators, are designed to cover specific applications. Similarly, computer programs often are written to translate the linear-programming output, usually too technical in nature, into meaningful managerial reports ready to be used by middle and top managers. In the next

* For an introduction to decision theory, the reader is referred to Howard Raiffa, *Decision Analysis—Introductory Lectures on Choices under Uncertainty*, Addison-Wesley, 1970.

section, dealing with the role of the computer in solving mathematical-programming models, we will discuss these issues more extensively.

D) Applying Sensitivity Analysis

One of the most useful characteristics of linear-programming codes is their capability to perform sensitivity analysis on the optimalsolutions obtained for the problem originally formulated. These postoptimum analyses are important for several reasons:

a) *Data Uncertainty*

Much of the information that is used in formulating the linear program is uncertain. Future production capacities and product demand, product specifications and requirements, cost and profit data, among other information, usually are evaluated through projections and average patterns, which are far from being known with complete accuracy. Therefore, it is often significant to determine how sensitive the optimal solutionis to changes in those quantities, and how the optimal solutionvaries when actual experience deviates from the values used in the original model.

b) *Dynamic Considerations*

Even if the data were known with complete certainty, we would still want to perform sensitivity analysis on the optimal solution to find out how the recommended courses ofaction should be modified after some time, when changes most probably will have taken place in the original specification of the problem. In other words, instead of getting merely a static solution to the problem, it is usually desirable to obtain at least some appreciation for a dynamic situation.

c) *Input Errors*

Finally, we want to inquire how errors we may have committed in the original formulation of the problem may affect the optimalsolution.

In general, the type of changes that are important to investigate are changes in the objective-function coefficients, in the righthand-side elements, and in the matrix coefficients. Further, it is sometimes necessary to evaluate the impact on the objective function of introducing new variables or new constraints into the problem. Although it is often impossible to assess all of these changes simultaneously, good linear-programming codes provide several means of obtaining pertinent information about the impact of these changes with a minimum of extra computational work.

Further discussions on this topic, including a description of the types of output generated by computer codes, are presented in the next section of this chapter.

E) Testing and implementing the Solution

The solution should be tested fully to ensure that the model clearly represents the real situation. We already have pointed out the importance of conducting sensitivity analysis as part of this testing effort. Should the solution be unacceptable, new refinements have to be incorporated into the model and new solutions obtained until the mathematical-programming model is adequate.

When testing is complete, the model can be implemented. Implementation usually means solving the model with real data to arrive at a decision or set of decisions. We can distinguish two kinds of implementation: a single or one-time use, such as a plant location problem; and continual or repeated use of the model, such as production planning or blending problems. In the latter case, routine systems should be established to provide input data to the linear-programming model, and to transform the output into operating instructions. Care must be taken to ensure that the model is flexible enough to allow for incorporating changes that take place in the real operating system.

5.3 THE ROLE OF THE COMPUTER

Solving even small mathematical-programming problems demands a prohibitive amount of computational effort, if undertaken manually. Thus, all practical applications of mathematical programming require the use of computers.

In this section we examine the role of digital computers in solving mathematical-programming models. The majority of our remarks will be directed to linear-programming models, since they have the most advanced computer support and are the models used most widely in practice.

The task of solving linear programs is greatly facilitated by the very efficient linear-programming codes that most computer manufacturers provide for their various computer models. In a matter of one or two days, a potential user can become familiar with the program instructions (control language or control commands) that have to be followed in order to solve a given linear-programming problem by means of a specific commercial code. The ample availability of good codes is one of the basic reasons for the increasing impact of mathematical programming on management decision-making.

Commercial mathematical-programming codes have experienced an interesting evolution during the last decade. At the beginning, they were simple and rigid computer programs capable of solving limited-size models. Today, they are sophisticated and highly flexible information-processing systems, with modules that handle incoming information, provide powerful computational algorithms, and report the model results in a way that is acceptable to a manager or an application-oriented user. Most of the advances experienced in the design of mathematical-programming codes have been made possible by developments in software and hardware technology, as well as break-throughs in mathematical-programming theory. The continual improvement in mathematical-programming computational capabilities is part of the trend to reduce running times and computational costs, to facilitate solutions of larger problems (by using network theory and large-scale system theory), and to extend the capabilities beyond the solution of linear-programming models into nonlinear and integer-programmingproblems.

The field mathematical-programming computer systems is changing rapidly and the number of computer codes currently available is extremely large. For these reasons, we will not attempt to cover the specific instructions of any linear-programming code in detail. Rather, we will concentrate on describing the basic features common to most good commercial systems, emphasizing those important concepts the user has to understand in order to make sound use of the currently existing codes.

Input Specifications

The first task the user is confronted with, when solving a linear-programming model by means of a commercial computer code, is the specification of the model structure and the input of the corresponding values of the parameters. There are several options available to the user in the performance of this task. We will examine some of the basic features common to most good codes in dealing with input descriptions.

In general, a linear-programming problem has the following structure:

$$\text{Maximize (or Minimize) } z = \sum_{j=1}^{n} c_j x_j, \tag{1}$$

subject to:

$$\sum_{j=1}^{n} a_{ij} x_j \; (\geq, =, \leq) \; b_i, \quad (i = 1, 2, \ldots, m), \tag{2}$$

$$x_j \geq 0, \qquad \text{for some or all } j, \tag{3}$$

$$x_j \text{ free}, \qquad \text{for some or all } j, \tag{4}$$

$$x_j \geq \ell_j, \qquad (\ell_j \neq 0), \qquad \text{for some or all } j, \tag{5}$$

$$x_j \leq u_j, \qquad (u_j \neq \infty), \qquad \text{for some or all } j. \tag{6}$$

This model formulation permits constraints to be specified either as ‘‘greater than or equal to’’ ($\geq$) inequalities, ‘‘less than or equal to’’ ($\leq$) inequalities, or simple equalities ($=$). It also allows for some variables to be nonnegative, some to be unconstrained in sign, and some to have lower and/or upper bounds.

a) *Reading the Data*

The most important elements the user has to specify in order to input the problem data are:

Names of the decision variables, constraints, objective function(s), and righthand side(s). Names have to be given to each of this model's elements so that they can be identified easily by the user. The names cannot exceed a given number of characters (normally 6 or 8), depending on the code being used. Moreover, it is important to adopt proper mnemonics in assigning names to these elements for easy identification of the actual meaning of the variables and constraints. For instance, instead of designating by X37 and X38 the production of regular and white-wall tires during the month of January, it would be better to designate them by RTJAN and WTJAN, respectively.

Numerical values of the parameters a_{ij}, b_i, *and* c_j. Only nonzero coefficients have to be designated. This leads to significant savings in the amount of information to be input, since a great percentage of coefficientsare zero in most linear-programming applications. The specification of a nonzero coefficient requires three pieces of information: the row name, the column name, and the numerical value corresponding to the coefficient.

Nature of the constraint relationship. The user should indicate whether a given constraint has a $=$, $\geq$, or $\leq$ relationship. Also, many systems permit the user to specify a range on a constraint consisting of both the $\geq$ and $\leq$ relationships.

Free and bounded variables. Almost all codes assume the variables to be nonnegative unless specified otherwise. This reduces the amount of data to be input, since in most practical applications the variables are indeed nonnegative. Thus the user need only specify those variables that are ‘‘free’’ (i.e., bounded neither from above nor from below), and those variables that have lower and upper bounds. The lower and upper bounds may be either positive or negative numbers.

Nature of the optimization command. Instructions have to establish whether the objective function is to be minimized or maximized. Moreover, most linear-programming codes allow the user to specify, if so desired, several objective functions and righthand sides. The code then proceeds to select one objective function and one righthand side at a time, in a sequence prescribed by the user, and the optimal solution is found for eachof the resulting linear programs. This is a practical and effective way to perform sensitivity analysis. Whenever several objective functions and righthand sides are input, the user should indicate which one of these elements to use for a given optimization run.

Title of problem and individual runs. It is useful to assign a title to the overall problem and to each of the sensitivity runs that might be performed. These titles have to be supplied externally by the user.

The information given to the computer is stored in the main memory core or, if the problem is extremely large, in auxiliary memory devices. Needless to say, the computational time required to solve a problem increases significantly whenever auxiliary memory is required, due to the added information-processing time. Within the state of the current computer technology, large computers (e.g., IBM 370/195) can handle problems with up to 4000 constraints without using special large-scale theoretical approaches.

b) *Retrieving a Summary of the Input Data*

Most codes permit the user to obtain summarized or complete information on the input data. This information is useful in identifying possible errors due to careless data specification, or to inappropriate model formulation. In addition, the input summary can be of assistance in detecting any special structure of the model that can be exploited for more efficient computational purposes.

The following are examples of types of summary information on the data input that are available in many codes:

Number of rows and columns in the coefficient matrix,

Number of righthand sides and objective functions,

Density of the coefficient matrix (i.e., the percentage of nonzero coefficients in the total number of coefficients),

Number of nonzero elements in each row and column,

Value of the largest and smallest element in the coefficient matrix, righthand side(s), and objective function(s),

Printout of all equations in expanded form,

Picture of the full coefficient matrix or a condensed form of the coefficient matrix.

c) *Scaling the Input Data*

The simultaneous presence of both large and small numbers in the matrix of coefficients should be avoided whenever possible, because it tends to create problems of numerical instability. This can be avoided by changing the unit of measure of a given variable (say, from lbs to 100 lbs, or from \$1000 to \$)—this change will reduce (or increase) the numerical values on a given column; or by dividing (or multiplying) a row by a constant, thus reducing (or increasing) the numerical values of the coefficients belonging to that row.

d) *Matrix Generators*

When dealing with large linear-programming models, the task of producing the original matrix of coefficients usually requires an extraordinary amount of human effort and elapsed time. Moreover, the chance of performing these tasks without errors is almost nil. For example, a problem consisting of 500 rows and 2000 variables, with a matrix density of 10 percent, has $500 \times 2000 \times 0.10 = 100{,}000$ nonzero coefficients that need to be input. As we indicated before, most linear-programming codes require three pieces of information to be specified when reporting a nonzero coefficient; these are the corresponding row name, column name, and coefficient value. Thus, in our example the user will need to input 300,000 elements. If the input operations are to be performed manually, the chance of absolute freedom from error is extremely low.

Moreover, in most practical applications, there is a great need for restructuring the matrix of coefficients due to dynamic changes in the problem under study, to model-formulation alternatives to be analyzed, and to sensitivity runs to be performed. A matrix generator helps address these difficulties by providing an effective mechanism to produce and revise the coefficient matrix.

A matrix generator is a computer program that allows the user to input the linear-programming data in a convenient form by prescribing only a minimum amount of information. Normally, it takes advantage of the repetitive structure that characterizes most large linear-programming models, where many of the coefficients are $+1, -1$, or zero. The matrix generator fills in those coefficients automatically, leaving to the user the task of providing only the input of those coefficients the program cannot anticipate or compute routinely. Multistage or multiperiod models have the tendency to repeat a given submatrix of coefficients several times. The matrix generator automatically duplicates this submatrix as many times as necessary to produce the final model structure. Matrix generators also provide consistency checks on input data to help detect possible sources of error.

Matrix generators also can be used to compute the values of some of the model coefficients directly from raw data provided by the user (e.g., the program might calculate the variable unit cost of an item by first evaluating the raw material costs, labor costs, and variable overhead). Moreover, the matrix generator can provide validity checks to ensure that no blunt errors have been introduced in the data. Simple checks consist of counting the number of nonzero coefficients in some or all of the rows andcolumns, in analyzing the signs of some critical variable coefficients,and so forth.

Sophisticated mathematical-programming codes have general-purpose matrix generators as part of their system. Instances are also quite common where the user can prepare his own special-purpose matrix generator to be used as part of the linear-programming computations.

Solution Techniques

There has been considerable concern in the development of mathematical-programming systems to reduce the computational time required to solve a linear-programming problem by improving the techniques used in the optimization stage of the process. This has been accomplished primarily by means of refinements in the matrix-inversion procedures (derived from matrix triangularization concepts), improvements in the path-selection criterion (by allowing multiple pricing of several columns to be considered as possible incoming variables), and use of more efficient presolution techniques (by providing *crashing* options to be used in order to obtain an initial feasible solution, or by permitting the user to specify starting columns in the initial basis).

Other important issues associated with the use computers in solving linear-programming problems are the presence of numerical errors and the tolerances allowed in defining the degree of accuracy sought in performing the computations. Most codes permit the user to exercise some options with regard to these issues. We will comment briefly on some of the most important features of those options.

A. *Reinversion*

Since the simplex method is a numeric computational procedure, it is inevitable for roundoff errors to be produced and accumulated throughout the sequence of iterations required to find the optimal solution. These errors could create infeasibility ornonoptimal conditions; that is, when the final solution is obtained and the corresponding values of the variables are substituted in the original set of constraints, either the solution might not satisfy the original requirements, or the pricing-out computations might generate some nonzero reduced costs for the basic variables.

To address these problems, most codes use the original values of the coefficient matrix to reinvert the basis periodically, thus maintaining its accuracy. The frequency of reinversion can be specified externally by the user; otherwise, it will be defined internally by the mathematical-programming system. Some computer codes fix the automatic reinversion frequency at a constant number of iterations performed; e.g., the basis is reinverted every 50 to 100 iterations. Other codes define the reinversion frequency as a multiple of the number of rows in the problem to be optimized; e.g., the basis is reinverted every 1.25 to $1.40m$ iterations, where m is the number of constraints.

Most codes compute a basis reinversion when the final solution has been obtained to assure its feasibility and optimality.

B. *Tolerances*

In order to control the degree of accuracy obtained in the numerical computations, most linear-programming codes permit the user to specify the tolerances he is willing to accept. Feasibility tolerances are designed to check whether or not the values of the nonbasic variables and the reduced cost of the basic variables are zero. Unless otherwise specified by the user, these checks involve internally set-up tolerance limits. Typical limits are -0.000001 and $+0.000001$.

There are also pivot rejection tolerances that prevent a coefficient very close to zero from becoming a pivot element in a simplex iteration. Again, 10^{-6} is a normal tolerance to use for this purpose, unless the user specifies his own limits.

C. *Errors*

As we have indicated before, errors usually are detected when the optimal values of the variables are substituted in theoriginal set of constraints. *Row errors* measure the difference between the computed values of the lefthand sides of the constraints, and the original righthand-side value, i.e., for equality constraints:

$$\sum_{j=1}^{n} a_{ij} x_j^0 - b_i = i\text{th row error},$$

where the x_j^0's are the corresponding optimal values ofthe variable. Similarly, there are *column errors,* which are calculated by performing similar computations with the dual problem, i.e., for a basic column:

$$\sum_{i=1}^{m} a_{ij} y_i^0 - c_i = j\text{th column error},$$

where the y_i^0's are the corresponding optimal values for thedual variables. Some codes have an internally determined or externally specified error frequency, which dictates how often errors are calculated and checked against the feasibility tolerances. If the computed errors exceed the prescribed tolerances, a *basis reinversion* is called for automatically. Restoring feasibility might demand a Phase I computation, or a dual simplex computation.

Many codes offer options to the user with regard to the output of the error calculations. The user might opt for a detailed printout of all column and row errors, or might be satisfied with just the maximum column and row error.

Whenever extra accuracy is required, some codes allow *double precision* to be used. This means that the space reserved in the computer for the number of significant figures to represent the value of each variable is doubled. Computing with the additional significant digits provides additional precision in the calculations required by the solution procedure.

Output Specifications

Once the optimal solution has been obtained, most computer codesprovide the user with a great amount of information that describes in detail the specific values of the optimal solution and itssensitivity to changes in some of the parameters of the original linear-programming problem. We will review some of the most important features associated with output specifications.

a) *Standard Output Reports*

Typical information produced by standard commercial codes might include:

Optimal value of the objective function. This isobtained by substituting the optimal values of the decision-variables in the original objective function. When the model has been constructed properly, the value of the objective function is a finite number. If no feasible solution exists, the computerprogram will indicate that the problem is infeasible, and no specific value of the objective function will be given. The remaining alternative occurs when the objective function can be increased (if we are maximizing) or decreased (if we are minimizing) indefinitely. In this situation, the program will report an unbounded solution, which is a clear indication in any practical application that an error has been made in the model formulation.

Optimal values of the decision variables. For each ofthe decision variables the program specifies its optimal value.Except for degenerate conditions, all the basic variables assume positive values. Invariably, all nonbasic variables have zero values. There are as many basic variables as constraints in the original problem.

Slacks and surpluses in the constraints. ''Less than or equal to'' ($\leq$) constraints might have positive slacks associated with them. Correspondingly, ''greater than or equal to'' ($\geq$) constraints might have positive surpluses associated with them. The program reports the amount of these slacks and surpluses in the optimal solution. Normally, a slackcorresponds to an unused amount of a given resource, and a surplus corresponds to an excess above a minimum requirement.

Shadow prices for constraints. The shadow price associated with a given constraint corresponds to the change in the objective function when the original righthand side of that constraint is increased by one unit. Shadow prices usually can be interpreted as marginal costs (if we are minimizing) or marginal profits (if we are maximizing). Constraints that have positive slacks or surpluses have zero shadow prices.

Reduced costs for decision variables. Reduced cost can be interpreted as the shadow prices corresponding to the nonnegativity constraints. All basic variables have zero reduced costs. The reduced cost associated with a nonbasic variable corresponds tothe change in the objective function whenever the corresponding value of the nonbasic variable is increased from 0 to 1.

Ranges on coefficients of the objective function. Ranges are given for each decision variable, indicating the lower and upper bound the cost coefficient of the variable can take without changing the current value of the variable in the optimalsolution.

Ranges on coefficients of the righthand side. Ranges are given for the righthand-side element of each constraint, indicating the lower and upper value the righthand-side coefficient of a given constraint can take without affecting the value of the shadow price associated with that constraint.

Variable transitions resulting from changes in the coefficients of the objective function. Whenever a coefficient of the objective function is changed beyond the range prescribed above, a change of the basis will take place. This element of the output report shows, for each variable, what variable will leave the basis and what new variable will enter the basis if the objective-function coefficient of the corresponding variable were to assume a value beyond its current range. If there is no variable to drop, the problem becomes unbounded.

Variable transitions resulting from changes in thecoefficient of the righthand side. Similarly, whenever a coefficient of the righthand side of a constraint is changed beyond the range prescribed above, a change in the current basis will occur. This portion of the report shows, for each constraint, which variable will leave the basis and which new variable will enter the basis if the righthand-side coefficient of the corresponding constraint were to assume a value beyond its current range. If there is no variable to enter, the problem becomes infeasible.

In the next section we present a simple illustration of these output reports.

b) *Sensitivity and Parametric Analyses*

In addition to providing the information just described, most codes allow the user to perform a variety of sensitivity and parametric runs, which permit an effective analysis of the changes resulting in the optimal solution when theoriginal specifications of the problem are modified. Quite often the user is not interested in obtaining a single solution to a problem, but wants to examine thoroughly a series of solutions to a number of cases. Some of the system features to facilitate this process are:

Multiple objective functions and righthand sides. We noticed before that provisions are made in many codes for the user to input several objective functions. The problem is solved with one objective function at a time, the optimal solution of oneproblem serving as an initial solution for the new problem. Sometimes only a few iterations are required to determine the new optimum, so that this process is quite effective. In this fashion, changes of the optimal solution with changes in the cost or revenue structure of the model can be assessed very rapidly. Similar options are available for processing the problem with one righthand side at a time, through a sequence of righthand sides provided by the user. Some codes use the dual simplex method for this purpose.

Parametric Variation. Another way to assess sensitivity analysis with regard to objective functions and righthand sides is to allow continuous changes to occur from a specified set of coefficients for the objective function or the righthand side to another specified set. This continuous parametrization exhaustively explores the pattern of the solution sensitivity in a very efficient manner. Some codes allow for a joint parametrization of cost coefficients and righthand-side elements.

Revisions of the original model formulation. Finally, many codes allow revisions to be incorporated in the model structure without requiring a complete reprocessing of the new problem. These revisions might effect changes in specific parameters of the model, as well as introduce new variables andnew constraints.

c) *Report generators*

Given the massive amount of highly technical information produced by a linear-programming code, it is most desirable to translate the output into a report directed to an application-oriented user. This is almost mandatory when using linear programming as an operational tool to support routine managerial decision-making. The most sophisticated mathematical-programming systems contain capabilities for the generation of general-purpose reports. Special-purpose reports easily can be programmed externally in any one of the high-level programming languages (like FORTRAN, APL, or BASIC).

5.4 A SIMPLE EXAMPLE

The basic purpose of this section is to illustrate, via a very small and simple example, the elements contained in a typical computer output of a linear-programming model. The example presented is a multistage production-planning problem. In order tosimplify our discussion and to facilitate the interpretation of the computer output, we have limited the size of the problem by reducing to a bare minimum the number of machines, products, and time periods being considered. This tends to limit the degree of realism of the problem, but greatly simplifies the model formulation. For some rich and realistic model-formulation examples, the reader is referred to the exercises at the end of this chapter, and to Chapters 6, 7, 10,and 14.

A Multistage Planning Problem

An automobile tire company has the ability to produce both nylon and fiberglass tires. During the next three months they have agreed to deliver tires as follows:

Date	*Nylon*	*Fiberglass*
June 30	4,000	1,000
July 31	8,000	5,000
August 31	3,000	5,000
Total	15,000	11,000

The company has two presses, a Wheeling machine and a Regal machine, and appropriate molds that can be used to produce these tires, with the following production hours available in the upcoming months:

	Wheeling machine	*Regal machine*
June	700	1500
July	300	400
August	1000	300

The production rates for each machine-and-tire combination, in terms of *hours per tire*, are as follows:

	Wheeling machine	*Regal machine*
Nylon	0.15	0.16
Fiberglass	0.12	0.14

The variable costs of producing tires are \$5.00 per operating hour, regardless of which machine is being used or which tire is being produced. There is also an inventory-carrying charge of \$0.10 per tire per month. Material costs for the nylon and fiberglass tires are \$3.10 and \$3.90 per tire, respectively. Finishing, packaging and shipping costs are \$0.23 per tire. Prices have been set at \$7.00 per nylon tire and \$9.00 per fiberglass tire.

The following questions have been raised by the production manager of the company:

a) How should the production be scheduled in order to meet the delivery requirements at minimum costs?
b) What would be the total contribution to be derived from this optimal schedule?
c) A new Wheeling machine is due to arrive at the beginning of September. For a \$200 fee, it would be possible to expedite the arrival of the machine to August 2, making available 172 additional hours of Wheeling machine time in August. Should the machine be expedited?
d) When would it be appropriate to allocate time for the yearly maintenance check-up of the two machines?

Model Formulation

We begin by applying the steps in model formulation recommended in Section 5.2.

Selection of a Time Horizon

In this particular situation the time horizon covers three months, divided into three time periods of a month's duration each. More realistic production-planning models normally have a full-year time horizon.

Selection of Decision Variables and Parameters

We can determine what decision variables are necessary by defining exactly what information the plant foreman must have in order to schedule the production. Essentially, he must know the number of each type of tire to be produced on each machine in each month and the number of each type of tire to place in inventory at the end of each month. Hence, we have the following decision variables:

$W_{n,t}$ = Number of nylon tires to be produced on the Wheeling machine during month t;
$R_{n,t}$ = Number of nylon tires to be produced on the Regal machine during month t;
$W_{g,t}$ = Number of fiberglass tires to be produced on the Wheeling machine in month t;
$R_{g,t}$ = Number of fiberglass tires to be produced on the Regal machine in month t;
$I_{n,t}$ = Number of nylon tires put into inventory at the end of month t;
$I_{g,t}$ = Number of fiberglass tires put into inventory at the end of month t.

In general there are six variables per time period and since there are three months under consideration, we have a total of eighteen variables. However, it should be clear that it would never be optimal to put tires into inventory at the end of August since all tires must be *delivered* by then. Hence, we can ignore the inventory variables for August.

The parameters of the problem are represented by the demand requirements, the machine availabilities, the machine productivity rates, and the cost and revenue information. All these parameters are assumed to be known deterministically.

Definition of the Constraints

There are two types of constraint in this problem representing production-capacity available and demand requirements at each month.

Let us develop the constraints for the month of June. The production-capacity constraints can be written in terms of production hours on each machine. For the Wheeling machine in June we have:

$$0.15W_{n,1} + 0.12W_{g,1} \leq 700,$$

while, for the Regal machine in June, we have:

$$0.16R_{\mathrm{n},1} + 0.14R_{\mathrm{g},1} \leq 1500.$$

The production constraints for future months differ only in the available number of hours of capacity for the righthand side.

Now consider the demand constraints for June. For each type of tire produced in June we must meet the demand requirement and then put any excess production into inventory. The demand constraint for nylon tires in June is then:

$$W_{\mathrm{n},1} + R_{\mathrm{n},1} - I_{\mathrm{n},1} = 4000,$$

while for fiberglass tires in June it is:

$$W_{\mathrm{g},1} + R_{\mathrm{g},1} - I_{\mathrm{g},1} = 1000.$$

In July, however, the tires put into inventory in June are available to meet demand. Hence, the demand constraint for nylon tires in July is:

$$I_{\mathrm{n},1} + W_{\mathrm{n},2} + R_{\mathrm{n},2} - I_{\mathrm{n},2} = 8000,$$

while for fiberglass tires in July it is:

$$I_{\mathrm{g},1} + W_{\mathrm{g},2} + R_{\mathrm{g},2} - I_{\mathrm{g},2} = 5000.$$

In August it is clear that tires will *not* be put into inventory at the end of the month, so the demand constraint for nylon tires in August is:

$$I_{\mathrm{n},2} + W_{\mathrm{n},3} + R_{\mathrm{n},3} = 3000,$$

while for fiberglass tires in August it is:

$$I_{\mathrm{g},2} + W_{\mathrm{g},3} + R_{\mathrm{g},3} = 5000.$$

Finally, we have the nonnegativity constraints on all of the decision variables:

$$\begin{array}{rlrl}
W_{\mathrm{n},t} \geq 0, & \qquad W_{\mathrm{g},t} \geq 0, & \quad (t = 1, 2, 3); \\
R_{\mathrm{n},t} \geq 0, & \qquad R_{\mathrm{g},t} \geq 0, & \quad (t = 1, 2, 3); \\
I_{\mathrm{n},t} \geq 0, & \qquad I_{\mathrm{g},t} \geq 0, & \quad (t = 1, 2).
\end{array}$$

Selection of the Objective Function

The total revenues to be obtained in this problem are fixed, because we are meeting all the demand requirements, and maximization of profit becomes equivalent to minimization of cost. Also, the material-cost component is fixed, since we know the total amount of each product to be produced during the model time horizon. Thus, a proper objective function to select is the minimization of the variable relevant cost components: variable production costs plus inventory-carrying costs.

Now, since each kind of tire on each machine has a different production rate, the cost of producing a tire on a particular machine will vary, even though the variable cost per hour is constant for each tire-and-machine combination. The variable production cost per tire for the fiberglass tires made on the Regal machine can be determined by multiplying the production rate (in hours/tire) by the variable production cost (in \$/hour) resulting in $(0.14)(5) = \$0.70$/tire. The remaining costs for producing each tire on each machine can be computed similarly, yielding

	Wheeling machine	*Regal machine*
Nylon	0.75	0.80
Fiberglass	0.60	0.70

Table 5.3 Multistage Planning Model*

			June						July						August						
			Nylon		*Glass*		*Inventory*		*Nylon*		*Glass*		*Inventory*		*Nylon*		*Glass*				
			$W_{n,1}$	$R_{n,1}$	$W_{g,1}$	$R_{g,1}$	$I_{n,1}$	$I_{g,1}$	$W_{n,2}$	$R_{n,2}$	$W_{g,2}$	$R_{g,2}$	$I_{n,2}$	$I_{g,2}$	$W_{n,3}$	$R_{n,3}$	$W_{g,3}$	$R_{g,3}$		RHS	
Machine time constraints	June	Wheeling	0.15		0.12														$\leq$	700	Hours available
		Regal		0.16		0.14													$\leq$	1500	
	July	Wheeling							0.15		0.12								$\leq$	300	
		Regal								0.16		0.14							$\leq$	400	
	Aug.	Wheeling													0.15		0.12		$\leq$	1000	
		Regal														0.16		0.14	$\leq$	300	
Demand constraints	June	Nylon	1	1			−1												=	4000	Number of tires demanded
		Glass			1	1		−1											=	1000	
	July	Nylon					1		1	1			−1						=	8000	
		Glass						1			1	1		−1					=	5000	
	Aug.	Nylon											1		1	1			=	3000	
		Glass												1			1	1	=	5000	
Objective			0.75	0.80	0.60	0.70	0.10	0.10	0.75	0.80	0.60	0.70	0.10	0.10	0.75	0.80	0.60	0.70		Minimum	

* All blanks are zeros.

Given the inventory-carrying cost of \$0.10 per tire per month, we have the following objective function for minimizing costs:

$$\sum_{t=1}^{3}(0.75W_{\text{n},t} + 0.80R_{\text{n},t} + 0.60W_{\text{g},t} + 0.70R_{\text{g},t} + 0.10I_{\text{n},t} + 0.10I_{\text{g},t});$$

and we understand that $I_{\text{n},3} = 0$ and $I_{\text{g},3} = 0$.

The formulation of this problem is summarized in Table 5.3. This problem is what we call a multistage model, because it contains more than one time period. Note that the constraints of one time period are linked to the constraints of another only by the inventory variables. This type of problem structure is very common in mathematical programming. Note that there are very few elements different from 0, 1, and -1 in the tableau given in Table 5.3. This problem structure can be exploited easily in the design of a matrix generator, to provide the input for the linear-programming computation.

Computer Results

We will now present the computer output obtained by solving the linear-programming model set forth in Table 5.3 by means of an interactive system operated from a computer terminal.
The notation describing the decision variables has been changed slightly, in order to facilitate computer printouts. For example:

WN–T = Number of nylon tires produced on the Wheeling machine in month T.

Similar interpretations can be given to variables WG–T (number of fiberglass tires on the Wheeling machine), RN–T, and RG–T (number of nylon and fiberglass tires, respectively, produced on the Regal machine at month T). IN–T and IG–T denote the number of nylon and fiberglass tires, respectively, left over at the end of period T.

The production constraints are represented by W–T and R–T, meaning the hours of Wheeling and Regal machine availability at period T. N–T and G–T stand for the demand at period T of nylon and fiberglass tires, respectively.

Figure 5.1 is the computer output of the problem. The reader should reflect about the meaning of each of the elements of the output. The output provides exactly the same information discussed under the title Standard Output Reports in Section 5.3. The reader is referred to that section for a detailed explanation of each output element.

Answering the Proposed Questions

With the aid of the optimal solution of the linear-programmingmodel, we can answer the questions that were formulated at the beginning of this problem.

a) *Production Scheduling*

Examination of the optimal values of the decision variables in thelinear-programming solution reveals that the appropriate production schedule should be:

		Wheeling machine	*Regal machine*
June	No. of nylon tires	1867	7633
	No. of fiberglass tires	3500	0
	Hrs. of unused capacity	0	279
July	No. of nylon tires	0	2500
	No. of fiberglass tires	2500	0
	Hrs. of unused capacity	0	0
August	No. of nylon tires	2667	333
	No. of fiberglass tires	5000	0
	Hrs. of unused capacity	0	247

The unused hours of each machine are the slack variables of the computer output.

The resulting inventory at the end of each month for the two types of products is as follows:

	June	*July*	*August*
Inventory of nylon tires	5500	0	0
Inventory of fiberglass tires	2500	0	0

```
TITLE: RUBICON RUBBER
PROCEED, DISPLAY, OR REJECT? PRO

MAXIMIZE OR MINIMIZE? MIN

OPTIMAL SOLUTION FOUND.
     COST          19173.3

OUTPUT OPTION? USUAL

ALL ITEMS NOT LISTED IN SECTIONS 1 - 4 HAVE THE VALUE ZERO.

*1* DECISION VARIABLES
  1. WN-1       1866.67
  2. RN-1       7633.33
  3. WG-1       3500.00
  5. IN-1       5500.00
  6. IG-1       2500.00
  8. RN-2       2500.00
  9. WG-2       2500.00
 13. WN-3       2666.67
 14. RN-3       333.333
 15. WG-3       5000.00

*2* SLACK(+) AND SURPLUS(-) IN CONSTRAINTS
  2. +R-1       278.667
  6. +R-3       246.667

*3* SHADOW PRICES FOR CONSTRAINTS
  1. W-1       -.333333
  3. W-2       -1.16667
  4. R-2       -.625000
  5. W-3       -.333333
  7. N-1        .800000
  8. G-1        .640000
  9. N-2        .900000
 10. G-2        .740000
 11. N-3        .800000
 12. G-3        .640000

*4* REDUCED COSTS FOR DECISION VARIABLES
  4. RG-1       .600000E-01
  7. WN-2       .250000E-01
 10. RG-2       8.71000
 11. IN-2       .200000
 12. IG-2       .200000
 16. RG-3       .600000E-01

OUTPUT OPTION? 5 7

*5* RANGES ON COEFFICIENTS OF OBJECTIVE COST
     VARIABLE  LOWER BOUND  CURRENT VALUE  UPPER BOUND
  1. WN-1        .67500        .75000        .77500
  2. RN-1        .75000        .80000        .87500
  3. WG-1        .58000        .60000        .66000
  4. RG-1        .64000        .70000       UNBOUNDED
  5. IN-1        .45714E-03    .10000        .12500
  6. IG-1        .80000E-01    .10000        8.8100
  7. WN-2        .72500        .75000       UNBOUNDED
  8. RN-2       UNBOUNDED      .80000        .89954
  9. WG-2       UNBOUNDED      .60000        .62000
 10. RG-2       -8.0100        .70000       UNBOUNDED
 11. IN-2       -.10000        .10000       UNBOUNDED
 12. IG-2       -.10000        .10000       UNBOUNDED
 13. WN-3        .67500        .75000        .80000
 14. RN-3        .75000        .80000        .87500
 15. WG-3       UNBOUNDED      .60000        .66000
 16. RG-3        .64000        .70000       UNBOUNDED
```

Figure 5.1 Computer printout of solution of the problem. (*Cont. on next page.*)

7 VARIABLE TRANSITIONS RESULTING FROM RANGING OBJECTIVE COST

	VARIABLE	LOWER BOUND VAR. IN	LOWER BOUND VAR. OUT	UPPER BOUND VAR. IN	UPPER BOUND VAR. OUT
1.	WN-1	RG-1	WG-1	WN-2	WN-1
2.	RN-1	+W-1	+R-1	RG-1	WG-1
3.	WG-1	WN-2	WN-1	RG-1	WG-1
4.	RG-1	RG-1	WG-1		
5.	IN-1	RG-2	+R-1	WN-2	WN-1
6.	IG-1	WN-2	WN-1	RG-2	+R-1
7.	WN-2	WN-2	WN-1		
8.	RN-2			RG-2	+R-1
9.	WG-2			WN-2	WN-1
1Ø.	RG-2	RG-2	+R-1		
11.	IN-2	IN-2	RN-3		
12.	IG-2	IG-2	RN-3		
13.	WN-3	RG-3	RN-3	+W-3	+R-3
14.	RN-3	+W-3	+R-3	RG-3	RN-3
15.	WG-3			RG-3	RN-3
16.	RG-3	RG-3	RN-3		

OUTPUT OPTION? 6 8

6 RANGES ON VALUES OF RIGHT-HAND-SIDE RHS1

	CONSTRNT	LOWER BOUND	CURRENT VALUE	UPPER BOUND
1.	W-1	438.75	7ØØ.ØØ	1845.Ø
2.	R-1	1221.3	15ØØ.Ø	UNBOUNDED
3.	W-2	38.75Ø	3ØØ.ØØ	6ØØ.ØØ
4.	R-2	121.33	4ØØ.ØØ	128Ø.Ø
5.	W-3	768.75	1ØØØ.Ø	1Ø5Ø.Ø
6.	R-3	53.333	3ØØ.ØØ	UNBOUNDED
7.	N-1	-3633.3	4ØØØ.Ø	5741.7
8.	G-1	-25ØØ.Ø	1ØØØ.Ø	3177.1
9.	N-2	25ØØ.Ø	8ØØØ.Ø	9741.7
1Ø.	G-2	25ØØ.Ø	5ØØØ.Ø	7177.1
11.	N-3	2666.7	3ØØØ.Ø	4541.7
12.	G-3	4583.3	5ØØØ.Ø	6927.1

8 VARIABLE TRANSITIONS RESULTING FROM RANGING RHS RHS1

	CONSTRNT	LOWER BOUND VAR. IN	LOWER BOUND VAR. OUT	UPPER BOUND VAR. IN	UPPER BOUND VAR. OUT
1.	W-1	+W-3	+R-1	+W-1	RN-1
2.	R-1	+W-3	+R-1		
3.	W-2	+W-3	+R-1	WN-2	IG-1
4.	R-2	+W-3	+R-1	RG-2	IN-1
5.	W-3	IN-2	+R-3	+W-3	RN-3
6.	R-3	IN-2	+R-3		
7.	N-1	+W-1	RN-1	+W-3	+R-1
8.	G-1	WN-2	WG-1	+W-3	+R-1
9.	N-2	RG-2	IN-1	+W-3	+R-1
1Ø.	G-2	WN-2	IG-1	+W-3	+R-1
11.	N-3	+W-3	RN-3	IN-2	+R-3
12.	G-3	+W-3	RN-3	IN-2	+R-3

OUTPUT OPTION? NO

PARAMETRICS? NO

OPTION? TER

Figure 5.1 (*Concluded*)

b) *Summary of Costs and Revenues*

Total costs. The total costs are the variable production costs, the inventory costs, the cost of raw materials, and the finishing, packaging, and shipping costs.

The variable production and inventory costs are obtained directly from the optimal value of the objective function of thelinear-programming model. These costs are equal to $19,173.30.

Raw-material costs are $3.10 per nylon tire and $3.90 per fiberglass tire. The total production of nylon and fiberglass tires is given in the delivery schedule. The total material costs are therefore:

Raw material cost for nylon tires	$3.10 \times 15{,}000 =$	$46,500
Raw material cost for fiberglass tires	$3.90 \times 11{,}000 =$	42,900
Total raw material cost		$89,400

The finishing, packaging, and shipping costs are $0.23 per tire. Since we are producing 26,000 tires, this cost amounts to $0.23 \times 26{,}000 = \$5{,}980$.

Thus the total costs are:

Variable production and inventory	$19,173.30
Raw material	89,400.00
Finishing, packaging and shipping	5,980.00
Total cost	$114,553.30

Total revenues. Prices per tire are $7.00 for nylon and $9.00 for fiberglass. Therefore, the total revenue is:

Nylon revenues	$\$7.00 \times 15{,}000 =$	$105,000
Fiberglass revenues	$\$9.00 \times 11{,}000 =$	99,000
Total revenues		$204,000

Contribution. The contribution to the company is:

Total revenues	$204,000.00
Total cost	114,553.30
Total contribution	$ 89,446.70

From this contribution we should subtract the corresponding overhead cost to be absorbed by this contract, in order to obtain the *net contribution before taxes* to the company.

c) *Expediting of the New Machine*

The new machine is a Wheeling machine, which is preferred to the Regal equipment. The question is *can* it be used, and *how much*? Even if the machine were fully utilized, the hourly marginal cost would be:

$$\frac{\$200}{172 \text{ hrs}} = \$1.16/\text{hr}.$$

Examination of the shadow price for Wheeling machines in August reveals that it would be worth only $0.33 to have an additional hour of time on Wheeling equipment. We should therefore recommend *against* expediting the additional machine.

d) *Maintenance Schedule*

We are not told in the problem statement the amount of time required to service a given machine, or whether maintenance can just as well be performed later, or how much it would cost to do it at night and on weekends. We therefore cannot tell the maintenance department exactly what to do. We can, however, tell them that the Wheeling machines are completely used, but 278 hours are available on the Regal machines in June and 246 in August. These hours would be ''free.''

The shadow prices show that, by adjusting the production schedule, Wheeling machine time could be made available in June and August at a cost of \$0.33/hr. During June we can have, at this cost, a total of 261.25 hours (the difference between the current availability, 700 hours, and the lower bound of the range for the W–1 righthand-side coefficient, 438.75). During August we will have available at \$0.33/hr a total of 231.25 hours (the difference betweenthe current availability, 1000 hours, and the lower bound of the range for the W–3 righthand-side coefficient, 768.75).

EXERCISES

1. The Pearce Container Corporation manufactures glass containers for a variety of products including soft drinks, beer, milk, catsup and peanut butter. Pearce has two products that account for 34% of its sales: container Type 35 and container Type 42. Pearce is interested in using a linear program to allocate production of these two containers to the five plants at which they are manufactured. The following is the estimated demand for the two types of containers for each quarter of next year: (1 unit = 100,000 containers)

Type	*Plant*	*1st Qtr.*	*2nd Qtr.*	*3rd Qtr.*	*4th Qtr.*
35	1	1388	1423	1399	1404
35	2	232	257	256	257
35	3	661	666	671	675
35*	4	31	32	34	36
42	1	2842	2787	3058	3228
42	2	2614	2551	2720	2893
42	3	1341	1608	1753	1887
42	4	1168	1165	1260	1305
42	5	1106	1138	1204	1206

* Plant 5 does not produce Type 35.

Pearce has ten machines at the five plants; eight of the machines can produce both Types 35 and 42, but two of the machines can produce only Type 42. Because the ten machines were purchased singly over a long period of time, no two machines have the same production rate or variable costs. After considerable research, the following information was gathered regarding the production of 1 unit (100,000 containers) by each of the ten machines:

Plant	*Machine*	*Type*	*Cost*	*Machine-days*	*Man-days*
1	D5	35 42	760 454	0.097 0.037	0.0194 0.0037
	D6	35 42	879 476	0.103 0.061	0.0206 0.0088
	C4	35 42	733 529	0.080 0.068	0.0204 0.0083
2	T	42	520	0.043	0.0109
	U2	35 42	790 668	0.109 0.056	0.0145 0.0143
3	K4	35 42	758 509	0.119 0.061	0.0159 0.0129
	J6	35 42	799 521	0.119 0.056	0.0159 0.0118
	70	35 42	888 625	0.140 0.093	0.0202 0.0196
4	1	35 42	933 538	0.113 0.061	0.0100 0.0081
5	V1	42	503	0.061	0.0135

During production a residue from the glass containers is deposited on the glass machines; during the course of a year machines are required to be shut down in order to clean off the residue. However, four of the machines are relatively new and will not be required to be shut down at all during the year; these are machines: C4, D5, K4, and V1. The following table shows the production days available for each machine by quarters:

Machine	*1st Qtr.*	*2nd Qtr.*	*3rd Qtr.*	*4th Qtr.*
C4	88	89	89	88
D5	88	89	89	88
D6	72	63	58	65
U2	81	88	87	55
T	88	75	89	88
K4	88	89	89	88
J6	37	89	39	86
70	54	84	85	73
1	42	71	70	68
V1	88	89	89	88
Days in quarter	88	89	89	88

In order to meet demands most efficiently, Pearce ships products from plant to plant. This transportation process ensures that the most efficient machines will be used first. However, transportation costs must also be considered. The following table shows transportation costs for shipping one unit between the five plants; the cost is the same for Type 35 and Type 42; Type 35 is not shipped to or from Plant 5.

	Inter-plant transport (100,000 containers)				
	To Plant				
From Plant	*1*	*2*	*3*	*4*	*5*
1	—	226	274	933	357
2	226	—	371	1022	443
3	274	371	—	715	168
4	941	1032	715	—	715
5	357	443	168	715	—

It is possible to store Type 35 and Type 42 containers almost indefinitely without damage to the containers. However, there is limited storage space at the five plants. Also, due to storage demands for other products, the available space varies during each quarter. The space available for Types 35 and 42 is as follows:

	Inventory capacity (100,000 bottles = 1 unit)			
Plant	*1st Qtr.*	*2nd Qtr.*	*3rd Qtr.*	*4th Qtr.*
1	376	325	348	410
2	55	48	62	58
3	875	642	573	813
4	10	15	30	24
5	103	103	103	103

It was found that there was no direct cost for keeping the containers in inventory other than using up storage space. However, there were direct costs for handling items, i.e., putting in *and* taking out of inventory. The costs for handling containers was as follows by type and by plant for one unit:

	Handling costs (in $/unit)	
Plant	*Type 35*	*Type 42*
1	85	70
2	98	98
3	75	75
4	90	80
5	—	67

a) Apply the stages of model formulation discussed in Section 5.2 to the Pearce Container Corp. problem. Precisely interpret the decision variables, the constraints, and the objective function to be used in the linear-programming model.
b) Indicate how to formulate the linear program mathematically. It is not necessary to write out the entire initial tableau of the model.
c) Determine the number of decision variables and constraints involved in the model formulation.

2. The Maynard Wire Company was founded in 1931 primarily to capitalize on the telephone company's expanding need for high-quality color-coded wire. As telephone services were rapidly expanding at that time, the need for quality color-coded wire was also expanding. Since then, the Maynard Wire Company has produced a variety of wire coatings, other wire products, and unrelated molded-plastic components. Today a sizable portion of its business remains in specially coated wire. Maynard Wire has only one production facility for coated wire, located in eastern Massachusetts, and has sales over much of the northeastern United States.

Maynard Wire is an intermediate processor, in that it purchases uncoated wire in standard gauges and then applies the various coatings that its customers desire. Basically there are only two types of coatings requested—standard inexpensive plastic and the higher-quality Teflon. The two coatings then come in a variety of colors, achieved by putting special dyes in the basic coating liquid. Since changing the color of the coating during the production process is a simple task, Maynard Wire has essentially two basic products.

Planning at Maynard Wire is done on a quarterly basis, and for the next quarter the demands for each type of wire in tons per month are:

Product	*July*	*August*	*September*
Plastic coated	1200	1400	1300
Quality Teflon	800	900	1150

The production of each type of wire must then be scheduled to minimize the cost of meeting this demand.

The Production process at Maynard Wire is very modern and highly automated. The uncoated wire arrives in large reels, which are put on spindles at one end of the plant. The uncoated wire is continuously drawn off each successive reel over some traverse guides and through a coating bath containing either the standard plastic or the more expensive Teflon. The wire then is pulled through an extruder, so that the coating liquid adheres evenly to the wire, which then continues through a sequence of four electric drying ovens to harden the coating. Finally, the wire is reeled up on reels similar to those it arrived on. Different dyes are added to the coating liquid during the process to produce the various colors of wire ordered.

Maynard Wire has two, basically independent, wire trains within the plant, one engineered by the Kolbert Engineering Corporation and the other purchased secondhand from the now defunct Loomis Wire Company. Both the standard plastic and the quality Teflon types of wire can be produced on either process train. The production rates in tons per day are:

Process train	*Plastic*	*Quality Teflon*
Kolbert	40	35
Loomis	50	42

Producing the quality Teflon wire is a slower process due to the longer drying time required. The associated variable operating cost for the month of July in dollars per day are:

Process train	*Plastic*	*Quality Teflon*
Kolbert	100	102
Loomis	105	108

However, because each month the process trains must be shut down for scheduled maintenance, there are fewer days available for production than days in the month. The process-train availabilities in days per month are:

Process train	*July*	*August*	*September*
Kolbert	26	26	29
Loomis	28	27	30

Both types of wire may be stored for future delivery. Space is available in Maynard Wire's own warehouse, but only up to 100 tons. Additional space is available essentially without limit on a leased basis. The warehousing costs in dollars per ton between July and August are:

Product	*Warehouse*	*Leased*
Plastic	8.00	12.00
Quality Teflon	9.00	13.00

A linear program has been formulated and solved that minimizes the total discounted manufacturing and warehousing costs. Future manufacturing and warehousing costs have been discounted at approximately ten percent per month. The MPS input format*, picture, and solution of the model are presented (see Figs. E5.1 and E5.2). Also, there is a parametric righthand-side study that increases the demand for standard plastic-coated wire in September from 1300 to 1600 tons. Finally, there is a parametric cost run that varies the warehousing cost for quality Teflon-coated wire from \$8.00 to \$12.00.

Typical rows and columns of the linear program are defined as follows:

$*$ MPS stands for Mathematical Programming System. It is a software package that IBM has developed to solve general linear-programming models.

Rows

COST	Objective function
DCOST	Change row for PARAOBJ
1P	Demand for plastic-coated wire in month 1
1K	Process-train availability in month 1
2WS	Warehouse limitation in month 2

Columns

1K-P	Production of plastic-coated wire on Kolbert train in month 1
WP12	Warehousing plastic-coated wire from the end of month 1 to the end of month 2
LP12	Leasing warehouse space for plastic-coated wire from the end of month 1 to the end of month 2
RHS1	Righthand side
RHS2	Change column for PARARHS

a) Explain the optimal policy for Maynard Wire Company when the objective function is COST and the righthand side is RHS1.
b) What is the resulting production cost for the 300-ton incremental production of plastic-coated wire in month 3?
c) How does the marginal production cost of plastic-coated wire vary when its demand in month 3 is shifted from 1300 to 1600 tons?
d) How does the operating strategy vary when the warehousing cost for quality Teflon-coated wire shifts from $8.00 to $12.00 with the demand for plastic-coated wire in month 3 held at 1600 tons?

3. Toys, Inc., is a small manufacturing company that produces a variety of children's toys. In the past, water pistols have been an exceptionally profitable item, especially the miniature type which can be hidden in the palm of one hand. However, children recently have been buying water rifles, which permit the stream of water to be projected much farther. Recognizing that this industry trend was not a short-lived fad, Toys, Inc., started to produce a line of water rifles.

After several months of production, Toys' General Manager, Mr. Whett, ran into a storage problem. The older and smaller water pistols had not occupied much space, but the larger water rifles were quite bulky. Consequently, Mr. Whett was forced to rent storage space in a public warehouse at 28¢ per case per month, plus 44¢ per case for cost of handling. This made Mr. Whett wonder whether producing water rifles was profitable. In addition, Mr. Whett wondered whether it might not be better to increase his production capacity so that fewer cases would be stored in the slack season.

Data:

The following information was obtained from Toys' accounting department:

Desired return on investment: 10% after taxes

Tax rate: 55% (including state taxes)

Variable manufacturing cost: $21.00/case, or $9.50/case after taxes

(Variable costs include all overhead and so-called ‘‘fixed" costs, except for the cost of production equipment. This seems appropriate, since the company has expanded so rapidly that ‘‘fixed" costs have actually been variable.)

Warehousing: $0.28/case per month, or $0.126/case per month after taxes

Handling: $0.44/case, or $0.198/case after taxes

Opporturnity cost of tying up capital in inventory: ($21.00 $\times$ 10%) $\div$ 12 months = $0.18/case per month

Selling price: $28.10/case, or $12.61 after taxes

Existing production capacity: 2750 cases per month

Cost of additional production capacity: $6400/year after taxes, for each additional 1000 cases/month. This figure takes into account the investment tax credit and the discounted value of the tax shield arising out of future depreciation.

The anticipated demand, based on past sales data, for the next 12 months is given below. The first month is October, and the units are in cases.

Month	*Demand* (Cases)
October	1490
November	2106
December	2777
January	843
February	1105
March	2932
April	1901
May	4336
June	4578
July	1771
August	4745
September	3216
Total	31800

Formulating the model:

The program variables are defined as follows:

PRD-1 to PRD12 identify the *production* constraints for the 12 periods.

DEM-1 to DEM12 identify the *dem*and constraints for the 12 periods.

CAP-1 to CAP12 identify the *cap*acity constraints for the 12 periods.

CNG-1 to CNG12 identify the constraints describing cha*nges* in inventory levels for the 12 periods.

X1 to X12 are the cases produced in each period. Associated with these variables are production costs of $9.50.

S1 to S12 are the cases sold in each period. Associated with these variables are revenues of $12.61 per case.

Y1 to Y12 are the cases in inventory at the beginning of the designated period. Associated with these variables are storage and cost of capital charges, totaling $0.306/case per month.

U1 to U12 are the unfilled demand in each period. No attempt has been made to identify a penalty cost associated with these variables.

Q1 to Q11 are the changes in inventory levels from one period to the next. Since a handling charge of $0.198/case is associated only with increases in inventory, these variables have been further designated as Q1+, Q1−, etc. to indicate increases (+) and decreases (−) in inventory levels.

+CAP-1 to +CAP12 are the slack variables supplied by the computer to represent unused production capacity in the designated period.

A typical production constraint, PRD-2, is shown below:

$$\text{Y2} + \text{X2} - \text{S2} - \text{Y3} = 0.$$

This expression indicates that the beginning inventory, plus the production, minus the sales must equal the ending inventory.

In the beginning of period 1 and at the end of period 12, the inventory level is set equal to zero. Hence, these equations become:

$$\text{X1} - \text{S1} - \text{Y2} = 0 \qquad \text{and} \qquad \text{Y12} + \text{X12} - \text{S12} = 0.$$

A typical demand constraint, DEM-2, is shown below:

$$\text{S2} + \text{U2} = 2106.$$

This expression indicates that the cases sold, plus the unfilled demand, must equal the total demand for that period.

A typical capacity constraint, CAP-2, is shown below:

$$\text{X2} \leq 2750.$$

This inequality indicates that the maximum number of cases that can be produced in any given month is 2750.

And lastly, a typical inventory level constraint, CNG-2, is shown below:

$$Y3 - Y2 = Q2 = (Q2+) - (Q2-).$$

This expression indicates that Q2 must equal the change in inventory level that occurs during period 2.

Since there is no beginning inventory, the change in inventory level that occurs during period 1 must equal the beginning inventory for period 2. Hence,

$Y2 = Q1 +$ Q1– must be zero, since negative inventories are impossible.

The *objective function* is to maximize the contribution, which equals:

$$\$12.61 \sum_{i=1}^{12} Si \quad - 9.50 \sum_{i=1}^{12} Xi \quad - 0.306 \sum_{i=1}^{12} Yi \quad - 0.198 \sum_{i=1}^{12} Qi+$$

The following 9 pages provide the output to be used in discussing the Toys, Inc. problem.

Page 257 gives the optimum solution for the initial problem statement. Capacity is fixed at 2750 cases per month at every time period.

Pages 258 and 259 contain, respectively, the cost ranges and righthand-side ranges associated with the optimum solution of the initial problem.

Pages 260 through 264 give details pertaining to a parametric analysis of the capacity availability. In each time period, the capacity is increased from its original value of 2750 to 2750 + THETA × 1375. The computer reports only solutions corresponding to a change of basis. Such changes have taken place at values of THETA equal to 0.235, 0.499, 1.153, 1.329, and 1.450, which are reported on pages 260 to 264, respectively.

Page 265 presents a parametric study for the cost associated with unfilled demand. The cost of unfilled demand is increased from its initial value of 0 to PHI × 1, for every demand period. Page 257 gives the optimum solution for PHI = 0; page 265 provides the optimum solution for PHI = 0.454. Beyond this value of PHI the solution does not change.

COMPUTER INPUT

We first present the basic input of the linear programming model.

The ROWS listing provides the names given to every row in the linear programming model. The first two rows are, respectively, the original cost objective function and the elements to be added to the objective function later in order to perform sensitivity analysis. The first letter in each heading specifies the nature of the constraint represented by the corresponding row.

(N = unrestricted; E = equality; L = less-than-or-equal-to constraint; G = greater-than-or-equal-to constraint.)

Under the COLUMNS heading every nonzero coefficient is identified by indicating the column name, the row name, and the corresponding numerical value of the coefficient.

```
ROWS
 N   COST
 N   DCOST
 E   1P
 E   1Q
 L   1K
 L   1L
 E   2P
 E   2Q
 L   2K
 L   2L
 E   3P
 E   3Q
 L   3K
 L   3L
 L   2WS
 L   3WS
COLUMNS
     1K-P      COST       100.0
     1K-P      1P          40.0
     1K-P      1K           1.0
     1K-Q      COST       102.0
     1K-Q      1Q          35.0
     1K-Q      1K           1.0
     1L-P      COST       105.0
     1L-P      1P          50.0
     1L-P      1L           1.0
     1L-Q      COST       108.0
     1L-Q      1Q          42.0
     1L-Q      1L           1.0
     WP12      COST         8.0
     WP12      1P          -1.0
     WP12      2P           1.0
     WP12      2WS          1.0
     WP13      COST         8.0
     WP13      1P          -1.0
     WP13      3P           1.0
     WP13      2WS          1.0
     WP13      3WS          1.0
     WQ12      COST         9.0
     WQ12      DCOST        1.0
     WQ12      1Q          -1.0
     WQ12      2Q           1.0
     WQ12      2WS          1.0
     WQ13      COST         9.0
     WQ13      DCOST        1.0
     WQ13      1Q          -1.0
     WQ13      3Q           1.0
     WQ13      2WS          1.0
     WQ13      3WS          1.0
     LP12      COST        12.0
     LP12      1P          -1.0
     LP12      2P           1.0
     LP13      COST        14.0
     LP13      1P          -1.0
     LP13      3P           1.0
     LQ12      COST        13.0
     LQ12      1Q          -1.0
     LQ12      2Q           1.0
     LQ13      COST        15.0
     LQ13      1Q          -1.0
     LQ13      3Q           1.0
     2K-P      COST        90.0
     2K-P      2P          40.0
     2K-P      2K           1.0
     2K-Q      COST        92.0
     2K-Q      2Q          35.0
     2K-Q      2K           1.0
```

Figure E5.1 Model of program for manufacturing and warehousing costs. (*Continued on next page.*)

```
    2L-P      COST        95.0
    2L-P      2P          50.0
    2L-P      2L           1.0
    2L-Q      COST        98.0
    2L-Q      2Q          42.0
    2L-Q      2L           1.0
    WP23      COST         7.0
    WP23      2P          -1.0
    WP23      3P           1.0
    WP23      3WS          1.0
    WQ23      COST         8.0
    WQ23      DCOST        1.0
    WQ23      2Q          -1.0
    WQ23      3Q           1.0
    WQ23      3WS          1.0
    LP23      COST        11.0
    LP23      2P          -1.0
    LP23      3P           1.0
    LQ23      COST        12.0
    LQ23      2Q          -1.0
    LQ23      3Q           1.0
    3K-P      COST        80.0
    3K-P      3P          40.0
    3K-P      3K           1.0
    3K-Q      COST        82.0
    3K-Q      3Q          35.0
    3K-Q      3K           1.0
    3L-P      COST        85.0
    3L-P      3P          50.0
    3L-P      3L           1.0
    3L-Q      COST        88.0
    3L-Q      3Q          42.0
    3L-Q      3L           1.0
RHS
```

Under the RHS heading every nonzero right-hand-side value is given. In our case there are three different righthand-side values that will be presented one at a time to perform the required sensitivity analysis.

```
    RHS1      1P        1200.0
    RHS1      1Q         800.0
    RHS1      1K          26.0
    RHS1      1L          28.0
    RHS1      2P        1400.0
    RHS1      2Q         900.0
    RHS1      2K          26.0
    RHS1      2L          27.0
    RHS1      3P        1300.0
    RHS1      3Q        1150.0
    RHS1      3K          29.0
    RHS1      3L          30.0
    RHS1      2WS        100.0
    RHS1      3WS        100.0
    RHS2      3P           1.0
    RHS2      3P           1.0
    RHS3      1P        1200.0
    RHS3      1Q         800.0
    RHS3      1K          26.0
    RHS3      1L          28.0
    RHS3      2P        1400.0
    RHS3      2Q         900.0
    RHS3      2K          26.0
    RHS3      2L          27.0
    RHS3      3P        1600.0
    RHS3      3Q        1150.0
    RHS3      3K          29.0
    RHS3      3L          30.0
    RHS3      2WS        100.0
    RHS3      3WS        100.0
ENDATA
```

The ENDATA command instructs the computer program that all the required data has been specified.

Figure E5.1 (*Continued*)

CONTROL PROGRAM COMPILER

```
0001    PROGRAM
0002    TITLE('MAYNARD WIRE COMPANY')
0003    INITIALZ
0060    MOVE(XDATA,'MAYNARD')
0061    MOVE(XPBNAME,'PBFILE')
0062    CONVERT('SUMMARY')
0063    SETUP('MIN')
0064    MOVE(XOBJ,'COST')
0065    MOVE(XRHS,'RHS1')
0066    PICTURE
0067    PRIMAL
0068    SOLUTION
0069    RANGE
0070    MOVE(XCHCOL,'RHS2')
0071    XPARAM=0.0
0072    XPARDELT=50.0
0073    XPARMAX=300.0
0074    PARARHS
0075    SOLUTION
0076    MOVE(XRHS,'RHS3')
0077    MOVE(XCHROW,'DCOST')
0078    XPARAM=0.0
0079    XPARDELT=1.0
0080    XPARMAX=4.0
0081    PARAOBJ
0082    SOLUTION
0083    EXIT
0084    PEND
```

The CONTROL PROGRAM COMPILER represents the commands which are given to solve the problem with the data set specified before. They pertain to the type of analysis and output information to be obtained from the computer.

		1K-P	1K-Q	1L-P	1L-Q	WP1-2	WP1-3	WQ1-2	WQ1-3	LP1-2	LP1-3	LQ1-2	LQ1-3	2K-P	2K-Q	2L-P	2L-Q	WP2-3	WQ2-3	LP2-3	LQ2-3	3K-P	3K-Q	3L-P	3L-Q	RHS1	RHS2	RHS3
COST	N	B	C	C	C	A	A	A	A	B	B	B	B	B	B	B	B	A	A	B	B	B	B	B	B			
DCOST	N							1	1										1									
1P	E	B		B		-1	-1			-1	-1															D		D
1Q	E		B		B			-1	-1			-1	-1													C		C
1K	L	1	1																							B		B
1L	L			1	1																					B		B
2P	E					1				1				B		B		-1		-1						D		D
2Q	E							1				1			B		B		-1		-1					C		C
2K	L													1	1											B		B
2L	L															1	1									B		B
3P	E						1				1							1		1		B		B		D	1	D
3Q	E								1				1						1		1		B		B	D		D
3K	L																					1	1			B		B
3L	L																							1	1	B		B
2WS	L					1	1	1	1																	B		B
3WS	L						1		1									1	1							B		B

This exhibit is a pictorial representation of the initial tableau showing the position and magnitude of the nonzero coefficients in the tableau. The following conventions have been used:

FROM	NOTATION	TO
1	A	10
10	B	100
100	C	1000
1000	D	10000

Figure E5.1 (*Concluded*)

MAYNARD WIRE COMPANY

SECTION 1 - ROWS FOR: COST AND RHS 1

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	COST	BS	15013.12143	15013.12143-	NONE	NONE	1.00000
2	DCOST	BS	33.75000	33.75000-	NONE	NONE	.
3	1P	EQ	1200.00000	.	1200.00000	1200.00000	2.38800-
4	1Q	EQ	800.00000	.	800.00000	800.00000	2.91429-
5	1K	BS	19.02143	6.97857	NONE	26.00000	.
6	1L	UL	28.00000	.	NONE	28.00000	14.40000
7	2P	EQ	1400.00000	.	1400.00000	1400.00000	10.37500-
8	2Q	EQ	900.00000	.	900.00000	900.00000	11.91429-
9	2K	UL	26.00000	.	NONE	26.00000	325.00000
10	2L	UL	27.00000	.	NONE	27.00000	423.75000
11	3P	EQ	1300.00000	.	1300.00000	1300.00000	1.90800-
12	3Q	EQ	1150.00000	.	1150.00000	1150.00000	2.34286-
13	3K	BS	28.05714	.94286	NONE	29.00000	.
14	3L	UL	30.00000	.	NONE	30.00000	10.40000
15	2WS	BS	33.75000	66.25000	NONE	100.00000	.
16	3WS	BS	.	100.00000	NONE	100.00000	.

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
17	1K-P	LL	.	100.00000	.	NONE	4.48000
18	1K-Q	BS	19.02143	102.00000	.	NONE	.
19	1L-P	BS	24.00000	105.00000	.	NONE	.
20	1L-Q	BS	4.00000	108.00000	.	NONE	.
21	WP12	LL	.	8.00000	.	NONE	.01300
22	WP13	LL	.	8.00000	.	NONE	8.48000
23	WQ12	BS	33.75000	9.00000	.	NONE	.
24	WQ13	LL	.	9.00000	.	NONE	9.57143
25	LP12	LL	.	12.00000	.	NONE	4.01300
26	LP13	LL	.	14.00000	.	NONE	14.48000
27	LQ12	LL	.	13.00000	.	NONE	4.00000
28	LQ13	LL	.	15.00000	.	NONE	15.57143
29	2K-P	BS	1.25000	90.00000	.	NONE	.
30	2K-Q	BS	24.75000	92.00000	.	NONE	.
31	2L-P	BS	27.00000	95.00000	.	NONE	.
32	2L-Q	LL	.	98.00000	.	NONE	21.35000
33	WP23	LL	.	7.00000	.	NONE	15.46700
34	WQ23	LL	.	8.00000	.	NONE	17.57143
35	LP23	LL	.	11.00000	.	NONE	19.46700
36	LQ23	LL	.	12.00000	.	NONE	21.57143
37	3K-P	LL	.	80.00000	.	NONE	3.68000
38	3K-Q	BS	28.05714	82.00000	.	NONE	.
39	3L-P	BS	26.00000	85.00000	.	NONE	.
40	3L-Q	BS	4.00000	88.00000	.	NONE	.

Figure E5.2 Solution of program. (*Cont.*)

MAYNARD WIRE COMPANY

PARARHS OBJ = COST RHS = RHS1 CHCOL = RHS2 PARAM = .

TIME = 0.34 MINS.

ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE	PARAM VALUE
M 13	0	13	24	9.57143	15088.1	39.2857

SECTION 1 - ROWS FOR: COST AND RHS1 50.0+ RHS2 [1]

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	COST	BS.	15194.66429	15194.66429-	NONE	NONE	1.00000
2	DCOST	BS	42.75000	42.75000-	NONE	NONE	.
3	1P	EQ	1200.00000	.	1200.00000	1200.00000	2.38800-
4	1Q	EQ	800.00000	.	800.00000	800.00000	2.91429-
5	1K	BS	19.27857	6.72143	NONE	26.00000	.
6	1L	UL	28.00000	.	NONE	28.00000	14.40000
7	2P	EQ	1400.00000	.	1400.00000	1400.00000	10.37500-
8	2Q	EQ	900.00000	.	900.00000	900.00000	11.91429-
9	2K	UL	26.00000	.	NONE	26.00000	325.00000
10	2L	UL	27.00000	.	NONE	27.00000	423.75000
11	3P	EQ	1350.00000	.	1350.00000	1350.00000	9.94800-
12	3Q	EQ	1150.00000	.	1150.00000	1150.00000	11.91429-
13	3K	UL	29.00000	.	NONE	29.00000	335.00000
14	3L	UL	30.00000	.	NONE	30.00000	412.40000
15	2WS	BS	42.75000	57.25000	NONE	100.00000	.
16	3WS	BS	9.00000	91.00000	NONE	100.00000	.

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
17	1K-P	LL	.	100.00000	.	NONE	4.48000
18	1K-Q	BS	19.27857	102.00000	.	NONE	.
19	1L-P	BS	24.00000	105.00000	.	NONE	.
20	1L-Q	BS	4.00000	108.00000	.	NONE	.
21	WP12	LL	.	8.00000	.	NONE	.01300
22	WP13	LL	.	8.00000	.	NONE	.44000
23	WQ12	BS	33.75000	9.00000	.	NONE	.
24	WQ13	BS	9.00000	9.00000	.	NONE	.
25	LP12	LL	.	12.00000	.	NONE	4.01300
26	LP13	LL	.	14.00000	.	NONE	6.44000
27	LQ12	LL	.	13.00000	.	NONE	4.00000
28	LQ13	LL	.	15.00000	.	NONE	6.00000
29	2K-P	BS	1.25000	90.00000	.	NONE	.
30	2K-Q	BS	24.75000	92.00000	.	NONE	.
31	2L-P	BS	27.00000	95.00000	.	NONE	.
32	2L-Q	LL	.	98.00000	.	NONE	21.35000
33	WP23	LL	.	7.00000	.	NONE	7.42700
34	WQ23	LL	.	8.00000	.	NONE	8.00000
35	LP23	LL	.	11.00000	.	NONE	11.42700
36	LQ23	LL	.	12.00000	.	NONE	12.00000
37	3K-P	LL	.	80.00000	.	NONE	17.08000
38	3K-Q	BS	29.00000	82.00000	.	NONE	.
39	3L-P	BS	27.00000	85.00000	.	NONE	.
40	3L-Q	BS	3.00000	88.00000	.	NONE	.

1 the reader should refer to the picture of the initial tableau to visualize the type of analysis being performed (What is the resulting righthand-side of the problem formed by RHS1 + 50 RHS2?)

Figure E5.2 Solution of program.

MAYNARD WIRE COMPANY

SECTION 1 - ROWS FOR: COST AND RHS1 + 100.0 RHS2

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	COST	BS	15692.06429	15692.06429-	NONE	NONE	1.00000
2	DCOST	BS	84.75000	84.75000-	NONE	NONE	.
3	1P	EQ	1200.00000	.	1200.00000	1200.00000	2.38800-
4	1Q	EQ	800.00000	.	800.00000	800.00000	2.91429-
5	1K	BS	20.47857	5.52143	NONE	26.00000	.
6	1L	UL	28.00000	.	NONE	28.00000	14.40000
7	2P	EQ	1400.00000	.	1400.00000	1400.00000	10.37500-
8	2Q	EQ	900.00000	.	900.00000	900.00000	11.91429-
9	2K	UL	26.00000	.	NONE	26.00000	325.00000
10	2L	UL	27.00000	.	NONE	27.00000	423.75000
11	3P	EQ	1400.00000	.	1400.00000	1400.00000	9.94800-
12	3Q	EQ	1150.00000	.	1150.00000	1150.00000	11.91429-
13	3K	UL	29.00000	.	NONE	29.00000	335.00000
14	3L	UL	30.00000	.	NONE	30.00000	412.40000
15	2WS	BS	84.75000	15.25000	NONE	100.00000	.
16	3WS	BS	51.00000	49.00000	NONE	100.00000	.

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
17	1K-P	LL	.	100.00000	.	NONE	4.48000
18	1K-Q	BS	20.47857	102.00000	.	NONE	.
19	1L-P	BS	24.00000	105.00000	.	NONE	.
20	1L-Q	BS	4.00000	108.00000	.	NONE	.
21	WP12	LL	.	8.00000	.	NONE	.01300
22	WP13	LL	.	8.00000	.	NONE	.44000
23	WQ12	BS	33.75000	9.00000	.	NONE	.
24	WQ13	BS	51.00000	9.00000	.	NONE	.
25	LP12	LL	.	12.00000	.	NONE	4.01300
26	LP13	LL	.	14.00000	.	NONE	6.44000
27	LQ12	LL	.	13.00000	.	NONE	4.00000
28	LQ13	LL	.	15.00000	.	NONE	6.00000
29	2K-P	BS	1.25000	90.00000	.	NONE	.
30	2K-Q	BS	24.75000	92.00000	.	NONE	.
31	2L-P	BS	27.00000	95.00000	.	NONE	.
32	2L-Q	LL	.	98.00000	.	NONE	21.35000
33	WP23	LL	.	7.00000	.	NONE	7.42700
34	WQ23	LL	.	8.00000	.	NONE	8.00000
35	LP23	LL	.	11.00000	.	NONE	11.42700
36	LQ23	LL	.	12.00000	.	NONE	12.00000
37	3K-P	LL	.	80.00000	.	NONE	17.08000
38	3K-Q	BS	29.00000	82.00000	.	NONE	.
39	3L-P	BS	28.00000	85.00000	.	NONE	.
40	3L-Q	BS	2.00000	88.00000	.	NONE	.

Figure E5.2 (*Cont.*)

MAYNARD WIRE COMPANY

PARARHS OBJ = COST RHS = RHS1 CHCOL = RHS2 PARAM = 100.00000

TIME = 0.41 MINS.

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE	PARAM VALUE
M	14	0	15	27	4.00000	15872.7	118.155

SECTION 1 - ROWS FOR: COST AND RHS1 + 100.0 RHS2

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	COST	BS	16296.46429	16296.46429-	NONE	NONE	1.00000
2	DCOST	BS	100.00000	100.00000-	NONE	NONE	.
3	1P	EQ	1200.00000	.	1200.00000	1200.00000	2.38800-
4	1Q	EQ	800.00000	.	800.00000	800.00000	2.91429-
5	1K	BS	21.67857	4.32143	NONE	26.00000	.
6	1L	UL	28.00000	.	NONE	28.00000	14.40000
7	2P	EQ	1400.00000	.	1400.00000	1400.00000	13.87500-
8	2Q	EQ	900.00000	.	900.00000	900.00000	15.91429-
9	2K	UL	26.00000	.	NONE	26.00000	465.00000
10	2L	UL	27.00000	.	NONE	27.00000	598.75000
11	3P	EQ	1450.00000	.	1450.00000	1450.00000	13.30800-
12	3Q	EQ	1150.00000	.	1150.00000	1150.00000	15.91429-
13	3K	UL	29.00000	.	NONE	29.00000	475.00000
14	3L	UL	30.00000	.	NONE	30.00000	580.40000
15	2WS	UL	100.00000	.	NONE	100.00000	4.00000
16	3WS	BS	93.00000	7.00000	NONE	100.00000	

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
17	1K-P	LL	.	100.00000	.	NONE	4.48000
18	1K-Q	BS	21.67857	102.00000	.	NONE	.
19	1L-P	BS	24.00000	105.00000	.	NONE	.
20	1L-Q	BS	4.00000	108.00000	.	NONE	.
21	WP12	LL	.	8.00000	.	NONE	.51300
22	WP13	LL	.	8.00000	.	NONE	1.08000
23	WQ12	BS	7.00000	9.00000	.	NONE	.
24	WQ13	BS	93.00000	9.00000	.	NONE	.
25	LP12	LL	.	12.00000	.	NONE	.51300
26	LP13	LL	.	14.00000	.	NONE	3.08000
27	LQ12	BS	26.75000	13.00000	.	NONE	.
28	LQ13	LL	.	15.00000	.	NONE	2.00000
29	2K-P	BS	1.25000	90.00000	.	NONE	.
30	2K-Q	BS	24.75000	92.00000	.	NONE	.
31	2L-P	BS	27.00000	95.00000	.	NONE	.
32	2L-Q	LL	.	98.00000	.	NONE	28.35000
33	WP23	LL	.	7.00000	.	NONE	7.56700
34	WQ23	LL	.	8.00000	.	NONE	8.00000
35	LP23	LL	.	11.00000	.	NONE	11.56700
36	LQ23	LL	.	12.00000	.	NONE	12.00000
37	3K-P	LL	.	80.00000	.	NONE	22.68000
38	3K-Q	BS	29.00000	82.00000	.	NONE	.
39	3L-P	BS	29.00000	85.00000	.	NONE	.
40	3L-Q	BS	1.00000	88.00000	.	NONE	.

Figure E5.2 (*Cont.*)

MAYNARD WIRE COMPANY

PARARHS OBJ = COST RHS = RHS1 CHCOL = RHS2 PARAM = 150.00000

TIME = 0.45 MINS.

ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE	PARAM VALUE
M 15	0	16	28	2.00000	16407.4	158.333

SECTION 1 - ROWS FOR: COST AND RHS 1 + 200.0 RHS 2

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	COST	BS	17031.86429	17031.86429-	NONE	NONE	1.00000
2	DCOST	BS	100.00000	100.00000-	NONE	NONE	.
3	1P	EQ	1200.00000	.	1200.00000	1200.00000	2.38800-
4	1Q	EQ	800.00000	.	800.00000	800.00000	2.91429-
5	1K	BS	22.87857	3.12143	NONE	26.00000	.
6	1L	UL	28.00000	.	NONE	28.00000	14.40000
7	2P	EQ	1400.00000	.	1400.00000	1400.00000	13.87500-
8	2Q	EQ	900.00000	.	900.00000	900.00000	15.91429-
9	2K	UL	26.00000	.	NONE	26.00000	465.00000
10	2L	UL	27.00000	.	NONE	27.00000	598.75000
11	3P	EQ	1500.00000	.	1500.00000	1500.00000	14.98800-
12	3Q	EQ	1150.00000	.	1150.00000	1150.00000	17.91429-
13	3K	UL	29.00000	.	NONE	29.00000	545.00000
14	3L	UL	30.00000	.	NONE	30.00000	664.40000
15	2WS	UL	100.00000	.	NONE	100.00000	4.00000
16	3WS	UL	100.00000	.	NONE	100.00000	2.00000

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
17	1K-P	LL	.	100.00000	.	NONE	4.48000
18	1K-Q	BS	22.87857	102.00000	.	NONE	.
19	1L-P	BS	24.00000	105.00000	.	NONE	.
20	1L-Q	BS	4.00000	108.00000	.	NONE	.
21	WP12	LL	.	8.00000	.	NONE	.51300
22	WP13	LL	.	8.00000	.	NONE	1.40000
23	WQ12	BS	.	9.00000	.	NONE	.
24	WQ13	BS	100.00000	9.00000	.	NONE	.
25	LP12	LL	.	12.00000	.	NONE	.51300
26	LP13	LL	.	14.00000	.	NONE	1.40000
27	LQ12	BS	33.75000	13.00000	.	NONE	.
28	LQ13	BS	35.00000	15.00000	.	NONE	.
29	2K-P	BS	1.25000	90.00000	.	NONE	.
30	2K-Q	BS	24.75000	92.00000	.	NONE	.
31	2L-P	BS	27.00000	95.00000	.	NONE	.
32	2L-Q	LL	.	98.00000	.	NONE	28.35000
33	WP23	LL	.	7.00000	.	NONE	7.88700
34	WQ23	LL	.	8.00000	.	NONE	8.00000
35	LP23	LL	.	11.00000	.	NONE	9.88700
36	LQ23	LL	.	12.00000	.	NONE	10.00000
37	3K-P	LL	.	80.00000	.	NONE	25.48000
38	3K-Q	BS	29.00000	82.00000	.	NONE	.
39	3L-P	BS	30.00000	85.00000	.	NONE	.
40	3L-Q	BS	.	88.00000	.	NONE	.

Figure E5.2 (*Cont.*)

MAYNARD WIRE COMPANY

PARARHS OBJ = COST RHS = RHS1 CHCOL = RHS2 PARAM = 200.00000

TIME = 0.49 MINS.

ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE	PARAM VALUE
M 16	0	40	37	25.4800	17031.9	200.000

SECTION 1 – ROWS FOR: COST AND RHS 1 + 250.0 RHS 2

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	COST	BS	17813.11429	17813.11429-	NONE	NONE	1.00000
2	DCOST	BS	100.00000	100.00000-	NONE	NONE	.
3	1P	EQ	1200.00000	.	1200.00000	1200.00000	2.38800-
4	1Q	EQ	800.00000	.	800.00000	800.00000	2.91429-
5	1K	BS	24.12857	1.87143	NONE	26.00000	.
6	1L	UL	28.00000	.	NONE	28.00000	14.40000
7	2P	EQ	1400.00000	.	1400.00000	1400.00000	13.87500-
8	2Q	EQ	900.00000	.	900.00000	900.00000	15.91429-
9	2K	UL	26.00000	.	NONE	26.00000	465.00000
10	2L	UL	27.00000	.	NONE	27.00000	598.75000
11	3P	EQ	1550.00000	.	1550.00000	1550.00000	15.62500-
12	3Q	EQ	1150.00000	.	1150.00000	1150.00000	17.91429-
13	3K	UL	29.00000	.	NONE	29.00000	545.00000
14	3L	UL	30.00000	.	NONE	30.00000	696.25000
15	2WS	UL	100.00000	.	NONE	100.00000	4.00000
16	3WS	UL	100.00000	.	NONE	100.00000	2.00000

SECTION 2 – COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
17	1K-P	LL	.	100.00000	.	NONE	4.48000
18	1K-Q	BS	24.12857	102.00000	.	NONE	.
19	1L-P	BS	24.00000	105.00000	.	NONE	.
20	1L-Q	BS	4.00000	108.00000	.	NONE	.
21	WP12	LL	.	8.00000	.	NONE	.51300
22	WP13	LL	.	8.00000	.	NONE	.76300
23	WQ12	BS	.	9.00000	.	NONE	.
24	WQ13	BS	100.00000	9.00000	.	NONE	.
25	LP12	LL	.	12.00000	.	NONE	.51300
26	LP13	LL	.	14.00000	.	NONE	.76300
27	LQ12	BS	35.75000	13.00000	.	NONE	.
28	LQ13	BS	78.75000	15.00000	.	NONE	.
29	2K-P	BS	1.25000	90.00000	.	NONE	.
30	2K-Q	BS	24.75000	92.00000	.	NONE	.
31	2L-P	BS	27.00000	95.00000	.	NONE	.
32	2L-Q	LL	.	98.00000	.	NONE	28.35000
33	WP23	LL	.	7.00000	.	NONE	7.25000
34	WQ23	LL	.	8.00000	.	NONE	8.00000
35	LP23	LL	.	11.00000	.	NONE	9.25000
36	LQ23	LL	.	12.00000	.	NONE	10.00000
37	3K-P	BS	1.25000	80.00000	.	NONE	.
38	3K-Q	BS	27.75000	82.00000	.	NONE	.
39	3L-P	BS	30.00000	85.00000	.	NONE	.
40	3L-Q	LL	.	88.00000	.	NONE	31.85000

Figure E5.2 (*Cont.*)

MAYNARD WIRE COMPANY

SECTION 1 - ROWS FOR: COST AND RHS 1 + 300.0 RHS2

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	COST	BS	18594.36429	18594.36429-	NONE	NONE	1.00000
2	DCOST	BS	100.00000	100.00000-	NONE	NONE	.
3	1P	EQ	1200.00000	.	1200.00000	1200.00000	2.38800-
4	1Q	EQ	800.00000	.	800.00000	800.00000	2.91429-
5	1K	BS	25.37857	.62143	NONE	26.00000	.
6	1L	UL	28.00000	.	NONE	28.00000	14.40000
7	2P	EQ	1400.00000	.	1400.00000	1400.00000	13.87500-
8	2Q	EQ	900.00000	.	900.00000	900.00000	15.91429-
9	2K	UL	26.00000	.	NONE	26.00000	465.00000
10	2L	UL	27.00000	.	NONE	27.00000	598.75000
11	3P	EQ	1600.00000	.	1600.00000	1600.00000	15.62500-
12	3Q	EQ	1150.00000	.	1150.00000	1150.00000	17.91429-
13	3K	UL	29.00000	.	NONE	29.00000	545.00000
14	3L	UL	30.00000	.	NONE	30.00000	696.25000
15	2WS	UL	100.00000	.	NONE	100.00000	4.00000
16	3WS	UL	100.00000	.	NONE	100.00000	2.00000

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
17	1K-P	LL	.	100.00000	.	NONE	4.48000
18	1K-Q	BS	25.37857	102.00000	.	NONE	.
19	1L-P	BS	24.00000	105.00000	.	NONE	.
20	1L-Q	BS	4.00000	108.00000	.	NONE	.
21	WP12	LL	.	8.00000	.	NONE	.51300
22	WP13	LL	.	8.00000	.	NONE	.76300
23	WQ12	BS	.	9.00000	.	NONE	.
24	WQ13	BS	100.00000	9.00000	.	NONE	.
25	LP12	LL	.	12.00000	.	NONE	.51300
26	LP13	LL	.	14.00000	.	NONE	.76300
27	LQ12	BS	33.75000	13.00000	.	NONE	.
28	LQ13	BS	122.50000	15.00000	.	NONE	.
29	2K-P	BS	1.25000	90.00000	.	NONE	.
30	2K-Q	BS	24.75000	92.00000	.	NONE	.
31	2L-P	BS	27.00000	95.00000	.	NONE	.
32	2L-Q	LL	.	98.00000	.	NONE	28.35000
33	WP23	LL	.	7.00000	.	NONE	7.25000
34	WQ23	LL	.	8.00000	.	NONE	8.00000
35	LP23	LL	.	11.00000	.	NONE	9.25000
36	LQ23	LL	.	12.00000	.	NONE	10.00000
37	3K-P	BS	2.50000	80.00000	.	NONE	.
38	3K-Q	BS	26.50000	82.00000	.	NONE	.
39	3L-P	BS	30.00000	85.00000	.	NONE	.
40	3L-Q	LL	.	88.00000	.	NONE	31.85000

Figure E5.2 (*Cont.*)

MAYNARD WIRE COMPANY

PARAOBJ OBJ = COST RHS = RHS3 CHROW = DCOST PARAM =

TIME = 0.57 MINS. Note: RHS3 = RHS1 + 300.0 RHS2

	ITER NUMBER	NUMBER NONOPT	VECTOR OUT	VECTOR IN	REDUCED COST	FUNCTION VALUE	PARAM VALUE
M	17	0	23	21	.51300	18645.7	.51300
M	18	0	24	22	.25000	18670.7	.76300

NO MAXIMUM FOR PARAMETER (ic. no basis change for increases in the parameter above this level)

SECTION 1 - ROWS FOR: COST + 4.0 DCOST AND RHS3

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1	COST	BS	18670.66429	18670.66429-	NONE	NONE	1.00000
2	DCOST	BS	.	.	NONE	NONE	4.00000
3	1P	EQ	1200.00000	.	1200.00000	1200.00000	2.38800-
4	1Q	EQ	800.00000	.	800.00000	800.00000	2.91429-
5	1K	BS	25.27857	.72143	NONE	26.00000	.
6	1L	UL	28.00000	.	NONE	28.00000	14.40000
7	2P	EQ	1400.00000	.	1400.00000	1400.00000	13.87500-
8	2Q	EQ	900.00000	.	900.00000	900.00000	15.91429-
9	2K	UL	26.00000	.	NONE	26.00000	465.00000
10	2L	UL	27.00000	.	NONE	27.00000	598.75000
11	3P	EQ	1600.00000	.	1600.00000	1600.00000	15.62500-
12	3Q	EQ	1150.00000	.	1150.00000	1150.00000	17.91429-
13	3K	UL	29.00000	.	NONE	29.00000	545.00000
14	3L	UL	30.00000	.	NONE	30.00000	696.25000
15	2WS	UL	100.00000	.	NONF	100.00000	3.48700
16	3WS	UL	100.00000	.	NONE	100.00000	1.75000

SECTION 2 - COLUMNS

NUMBER	.COLUMN.	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
17	1K-P	LL	.	100.00000	.	NONE	4.48000
18	1K-Q	BS	25.27857	102.00000	.	NONE	.
19	1L-P	BS	26.00000	105.00000	.	NONE	.
20	1L-Q	BS	2.00000	108.00000	.	NONE	.
21	WP12	BS	.	8.00000	.	NONE	.
22	WP13	BS	100.00000	8.00000	.	NONE	.
23	WQ12	LL	.	13.00000	.	NONE	3.48700
24	WQ13	LL	.	13.00000	.	NONE	3.23700
25	LP12	LL	.	12.00000	.	NONE	.51300
26	LP13	LL	.	14.00000	.	NONE	.76300
27	LQ12	BS	33.75000	13.00000	.	NONE	.
28	LQ13	BS	135.00000	15.00000	.	NONE	.
29	2K-P	BS	1.25000	90.00000	.	NONE	.
30	2K-Q	BS	24.75000	92.00000	.	NONE	.
31	2L-P	BS	27.00000	95.00000	.	NONE	.
32	2L-Q	LL	.	98.00000	.	NONE	28.35000
33	WP23	LL	.	7.00000	.	NONE	7.00000
34	WQ23	LL	.	12.00000	.	NONE	11.75000
35	LP23	LL	.	11.00000	.	NONE	9.25000
36	LQ23	LL	.	12.00000	.	NONE	10.00000
37	3K-P	BS	.	80.00000	.	NONE	.
38	3K-Q	BS	29.00000	82.00000	.	NONE	.
39	3L-P	BS	30.00000	85.00000	.	NONE	.
40	3L-Q	LL	.	88.00000	.	NONE	31.85000

Figure E5.2 (*Concluded*)

245800, BRADLEY TOYS, INC. PRODUCTION SCHEDULING

TOTAL ITERS	NO. ETAS	ETA REC	ROW IDENT.	CURRENT VALUE	CHOSEN VECTOR	VECTR REMVD	RHS NO.	C/V NO.	CURRENT D/J THETA/PHI	OPTIMAL PRI
54	54	0	OBJT1	90159.351			1		* * * *	

J(H)	BETA(H)	ROW(I)	PI(I)	B(I)
0 00000	90159.35199931	OBJT1	1.00000000	.
0 00000	466.00000000-	OBJT2	.	.
S1	1491.00000000	PRD-1	9.50000000-	.
S2	2106.00000000	PRD-2	9.50000000-	.
S3	2777.00000000	PRD-3	10.00400000-	.
S4	843.00000000	PRD-4	10.11200000-	.
S5	1105.00000000	PRD-5	10.41800000-	.
S6	2932.00000000	PRD-6	10.92200000-	.
S7	1901.00000000	PRD-7	11.03000000-	.
S8	4336.00000000	PRD-8	11.53400000-	.
S9	4578.00000000	PRD-9	11.84000000-	.
S10	1771.00000000	PRD10	11.94800000-	.
S11	4744.00000000	PRD11	12.45200000-	.
S12	2750.00000000	PRD12	12.61000000-	.
X1	1491.00000000	DEM-1	3.11000000	1491.00000000
X2	2343.00000000	DEM-2	3.11000000	2106.00000000
Q2+	237.00000000	DEM-3	2.60600000	2777.00000000
X4	2750.00000000	DEM-4	2.49800000	843.00000000
X5	2750.00000000	DEM-5	2.19200000	1105.00000000
Q5+	1645.00000000	DEM-6	1.68800000	2932.00000000
X7	2750.00000000	DEM-7	1.58000000	1901.00000000
Y7	3580.00000000	DEM-8	1.07600000	4336.00000000
Q4+	1907.00000000	DEM-9	.77000000	4578.00000000
X10	2750.00000000	DEM10	.66200000	1771.00000000
Y4	210.00000000	DEM11	.15800000	4744.00000000
U12	466.00000000	DEM12	.	3216.00000000
+ CAP-1	1259.00000000	+ CAP-1	.	2750.00000000
+ CAP-2	407.00000000	+ CAP-2	.	2750.00000000
X3	2750.00000000	+ CAP-3	.50400000	2750.00000000
Y10	1015.00000000	+ CAP-4	.61200000	2750.00000000
Q9-	1828.00000000	+ CAP-5	.91800000	2750.00000000
X6	2750.00000000	+ CAP-6	1.42200000	2750.00000000
Q8-	1586.00000000	+ CAP-7	1.53000000	2750.00000000
X8	2750.00000000	+ CAP-8	2.03400000	2750.00000000
X9	2750.00000000	+ CAP-9	2.34000000	2750.00000000
Q11-	1994.00000000	+ CAP10	2.44800000	2750.00000000
X11	2750.00000000	+ CAP11	2.95200000	2750.00000000
X12	2750.00000000	+ CAP12	3.11000000	2750.00000000
Y2	.	CNG-1	.10800000-	.
Y3	237.00000000	CNG-2	.19800000	.
Q3-	27.00000000	CNG-3	.	.
Y5	2117.00000000	CNG-4	.19800000	.
Y6	3762.00000000	CNG-5	.19800000	.
Q6-	182.00000000	CNG-6	.	.
Y8	4429.00000000	CNG-7	.19800000	.
Q7+	849.00000000	CNG-8	.	.
Y9	2843.00000000	CNG-9	.	.
Y11	1994.00000000	CNG10	.19800000	.
Q10+	979.00000000	CNG11	.	.

Figure E5.3 Optimum solution for Toys, Inc., cost and righthand-side ranges; parametric RHS analysis, and parametric cost ranging. (*Cont.*)

245800, BRADLEY TOYS, INC. PRODUCTION SCHEDULING

COST RANGES

BASIS VECTOR	BETA VALUE	COST IN PROBLEM	LIM 1	LIMIT 2	INCOMING VECTOR AT LIM 1	INCOMING VECTOR AT LIM 2
S1	1491.0000	-12.610000	* * * *	-9.5000000	UNBOUNDED	U1
S2	2106.0000	-12.610000	* * * *	-9.5000000	UNBOUNDED	U2
S3	2777.0000	-12.610000	* * * *	-10.004000	UNBOUNDED	U3
S4	842.99999	-12.610000	* * * *	-10.112000	UNBOUNDED	U4
S5	1105.0000	-12.610000	* * * *	-10.418000	UNBOUNDED	U5
S6	2932.0000	-12.610000	* * * *	-10.922000	UNBOUNDED	U6
S7	1901.0000	-12.610000	* * * *	-11.030000	UNBOUNDED	U7
S8	4336.0000	-12.610000	* * * *	-11.534000	UNBOUNDED	U8
S9	4578.0000	-12.610000	* * * *	-11.840000	UNBOUNDED	U9
S10	1771.0000	-12.610000	* * * *	-11.948000	UNBOUNDED	U10
S11	4744.0000	-12.610000	* * * *	-12.452000	UNBOUNDED	U11
S12	2750.0000	-12.610000	-12.758000	-9.5000000	Y12	+ CAP12
X1	1491.0000	9.5000000	9.1940000	12.610000	Q1+	U1
X2	2343.0000	9.5000000	9.3520000	9.6579999	Y12	U11
Q2+	237.00000	.19800000	.05000000	.35600000	Y12	U11
X4	2750.0000	9.5000000	* * * *	10.112000	UNBOUNDED	+ CAP-4
X5	2750.0000	9.5000000	* * * *	10.418000	UNBOUNDED	+ CAP-5
Q5+	1645.0000	.19800000		1.1160000	Q5-	+ CAP-5
X7	2750.0000	9.5000000	* * * *	11.030000	UNBOUNDED	+ CAP-7
Y7	3580.0000	.30600000	.15800000	.46400000	Y12	U11
Q4+	1907.0000	.19800000		.80999999	Q4-	+ CAP-4
X10	2750.0000	9.5000000	* * * *	11.948000	UNBOUNDED	+ CAP10
Y4	210.00000	.30600000	.15800000	.46400000	Y12	U11
U12	466.00000		-3.1100000	.14800000	+ CAP12	Y12
+ CAP-1	1259.0000		-3.1100000	.30600000	U1	Q1+
+ CAP-2	407.00000		-.15800000	.14800000	U11	Y12
X3	2750.0000	9.5000000	* * * *	10.004000	UNBOUNDED	+ CAP-3
Y10	1015.0000	.30600000	.15800000	.46400000	Y12	U11
Q9-	1828.0000		-.19800000	.77000000	Q9+	U9
X6	2750.0000	9.5000000	* * * *	10.922000	UNBOUNDED	+ CAP-6
Q8-	1586.0000		-.19800000	1.0760000	Q8+	U8
X8	2750.0000	9.5000000	* * * *	11.534000	UNBOUNDED	+ CAP-8
X9	2750.0000	9.5000000	* * * *	11.840000	UNBOUNDED	+ CAP-9
Q11-	1994.0000		-.19800000	.15800000	Q11+	U11
X11	2750.0000	9.5000000	* * * *	12.452000	UNBOUNDED	+ CAP11
X12	2750.0000	9.5000000	* * * *	12.610000	UNBOUNDED	+ CAP12
Y2		.30600000		* * * *	Q1+	UNBOUNDED
Y3	237.00000	.30600000	.15800000	.46400000	Y12	U11
Q3-	27.000000		-.19800000	2.6060000	Q3+	U3
Y5	2117.0000	.30600000	.15800000	.46400000	Y12	U11
Y6	3762.0000	.30600000	.15800000	.46400000	Y12	U11
Q6-	182.00000		-.19800000	1.6880000	Q6+	U6
Y8	4429.0000	.30600000	.15800000	.46400000	Y12	U11
Q7+	849.00000	.19800000		1.7280000	Q7-	+ CAP-7
Y9	2843.0000	.30600000	.15800000	.46400000	Y12	U11
Y11	1994.0000	.30600000	.15800000	.46400000	Y12	U11
Q10+	978.99999	.19800000		2.6460000	Q10-	+ CAP10

Figure E5.3 (*Cont.*)

245800, BRADLEY TOYS, INC. PRODUCTION SCHEDULING

RIGHT HAND SIDE RANGES

ROW NAME	CURRENT RHS VAL	PI VALUE	MINIMUM VALUE	MAXIMUM VALUE	OUTGOING VECTOR AT MIN	OUTGOING VECTOR AT MAX
PRD-1		-9.5000000	-1491.0000	1259.0000	X1	+ CAP-1
PRD-2		-9.5000000	-2343.0000	407.00000	X2	+ CAP-2
PRD-3		-10.004000	-27.000000	407.00000	Q3-	+ CAP-2
PRD-4		-10.112000	-210.00000	407.00000	Y4	+ CAP-2
PRD-5		-10.418000	-210.00000	407.00000	Y4	+ CAP-2
PRD-6		-10.922000	-182.00000	407.00000	Q6-	+ CAP-2
PRD-7		-11.030000	-210.00000	407.00000	Y4	+ CAP-2
PRD-8		-11.534000	-210.00000	407.00000	Y4	+ CAP-2
PRD-9		-11.840000	-210.00000	407.00000	Y4	+ CAP-2
PRD10		-11.948000	-210.00000	407.00000	Y4	+ CAP-2
PRD11		-12.452000	-210.00000	407.00000	Y4	+ CAP-2
PRD12		-12.610000	-466.00000	2750.0000	U12	S12
DEM-1	1491.000	3.1100000		2750.0000	S1	+ CAP-1
DEM-2	2106.000	3.1100000		2513.0000	S2	+ CAP-2
DEM-3	2777.000	2.6060000	2750.0000	3184.0000	Q3-	+ CAP-2
DEM-4	842.9999	2.4980000	632.99999	1250.0000	Y4	+ CAP-2
DEM-5	1105.000	2.1920000	895.00000	1512.0000	Y4	+ CAP-2
DEM-6	2932.000	1.6880000	2750.0000	3339.0000	Q6-	+ CAP-2
DEM-7	1901.000	1.5800000	1691.0000	2308.0000	Y4	+ CAP-2
DEM-8	4336.000	1.0760000	4126.0000	4743.0000	Y4	+ CAP-2
DEM-9	4578.000	.77000000	4368.0000	4985.0000	Y4	+ CAP-2
DEM10	1771.000	.66200000	1561.0000	2178.0000	Y4	+ CAP-2
DEM11	4744.000	.15800000	4534.0000	5150.9999	Y4	+ CAP-2
DEM12	3216.000		2750.0000	UNBOUNDED	U12	
+ CAP-1	2750.000		1491.0000	UNBOUNDED	+ CAP-1	
+ CAP-2	2750.000		2343.0000	UNBOUNDED	+ CAP-2	
+ CAP-3	2750.000	.50400000	2343.0000	2777.0000	+ CAP-2	Q3-
+ CAP-4	2750.000	.61200000	2343.0000	2960.0000	+ CAP-2	Y4
+ CAP-5	2750.000	.91800000	2343.0000	2960.0000	+ CAP-2	Y4
+ CAP-6	2750.000	1.4220000	2343.0000	2932.0000	+ CAP-2	Q6-
+ CAP-7	2750.000	1.5300000	2343.0000	2960.0000	+ CAP-2	Y4
+ CAP-8	2750.000	2.0340000	2343.0000	2960.0000	+ CAP-2	Y4
+ CAP-9	2750.000	2.3400000	2343.0000	2960.0000	+ CAP-2	Y4
+ CAP10	2750.000	2.4480000	2343.0000	2960.0000	+ CAP-2	Y4
+ CAP11	2750.000	2.9520000	2343.0000	2960.0000	+ CAP-2	Y4
+ CAP12	2750.000	3.1100000		3216.0000	S12	U12
CNG-1		-.10800000		237.00000	Y2	Q2+
CNG-2		.19800000	UNBOUNDED	237.00000		Q2+
CNG-3			-27.000000	UNBOUNDED	Q3-	
CNG-4		.19800000	UNBOUNDED	1907.0000		Q4+
CNG-5		.19800000	UNBOUNDED	1645.0000		Q5+
CNG-6			-182.00000	UNBOUNDED	Q6-	
CNG-7		.19800000	UNBOUNDED	849.00000		Q7+
CNG-8			-1586.0000	UNBOUNDED	Q8-	
CNG-9			-1828.0000	UNBOUNDED	Q9-	
CNG10		.19800000	UNBOUNDED	978.99999		Q10+
CNG11			-1994.0000	UNBOUNDED	Q11-	

Figure E5.3 (*Cont.*)

PARAMETRIC RIGHT-HAND-SIDE RANGING

245800, BRADLEY TOYS, INC. PRODUCTION SCHEDULING

TOTAL ITERS	NO. ETAS	ETA REC	ROW IDENT.	CURRENT VALUE	CHOSEN VECTOR	VECTR REMVD	RHS NO.	C/V NO.	CURRENT D/J THETA/PHI	CURRENT RT.I
62	62	0	OBJT1	94366.246	Q4-	Q4+	1	2	.23481818	

J(H)	BETA(H)	ROW(I)	PI(I)	B(I)+T*C(I)
0 00000	94366.24799940	OBJT1	1.00000000	.
0 00000	.	OBJT2	.	.
S1	1491.00000000	PRD-1	9.50000000-	.
S2	2106.00000000	PRD-2	9.50000000-	.
S3	2777.00000000	PRD-3	9.50000000-	.
S4	843.00000000	PRD-4	9.50000000-	.
S5	1105.00000000	PRD-5	9.80600000-	.
S6	2932.00000000	PRD-6	10.11200000-	.
S7	1901.00000000	PRD-7	10.41800000-	.
S8	4336.00000000	PRD-8	10.92200000-	.
S9	4578.00000000	PRD-9	11.22800000-	.
S10	1771.00000000	PRD10	11.33600000-	.
S11	4744.00000000	PRD11	11.84000000-	.
S12	3216.00000000	PRD12	12.14600000-	.
X1	1491.00000000	DEM-1	3.11000000	1491.00000000
X2	2106.00000000	DEM-2	3.11000000	2106.00000000
+ CAP-4	2229.87500000	DEM-3	3.11000000	2777.00000000
X4	843.00000000	DEM-4	3.11000000	843.00000000
X5	3072.87500000	DEM-5	2.80400000	1105.00000000
Q5+	1967.87500000	DEM-6	2.49800000	2932.00000000
X7	3072.87500000	DEM-7	2.19200000	1901.00000000
Y7	2108.75000000	DEM-8	1.68800000	4336.00000000
Q4-	.	DEM-9	1.38200000	4578.00000000
X10	3072.87500000	DEM10	1.27400000	1771.00000000
Y12	143.12500000	DEM11	.77000000	4744.00000000
Q2-	.	DEM12	.46400000	3216.00000000
+ CAP-1	1581.87500000	+ CAP-1	.	3072.87500000
+ CAP-2	966.87500000	+ CAP-2	.	3072.87500000
X3	2777.00000000	+ CAP-3	.	3072.87500000
Y10	512.37500000	+ CAP-4	.	3072.87500000
Q9-	1505.12500000	+ CAP-5	.30600000	3072.87500000
X6	3072.87500000	+ CAP-6	.61200000	3072.87500000
Q8-	1263.12500000	+ CAP-7	.91800000	3072.87500000
X8	3072.87500000	+ CAP-8	1.42200000	3072.87500000
X9	3072.87500000	+ CAP-9	1.72800000	3072.87500000
Q11-	1671.12500000	+ CAP10	1.83600000	3072.87500000
X11	3072.87500000	+ CAP11	2.34000000	3072.87500000
X12	3072.87500000	+ CAP12	2.64600000	3072.87500000
Y2	.	CNG-1	.30600000-	.
+ CAP-3	295.87500000	CNG-2	.	.
Q3+	.	CNG-3	.19800000	.
Y5	.	CNG-4	.19800000	.
Y6	1967.87500000	CNG-5	.19800000	.
Q6+	140.87500000	CNG-6	.19800000	.
Y8	3280.62500000	CNG-7	.19800000	.
Q7+	1171.87500000	CNG-8	.	.
Y9	2017.50000000	CNG-9	.	.
Y11	1814.25000000	CNG10	.19800000	.
Q10+	1301.87500000	CNG11	.	.

Figure E5.3 (*Cont.*)

245800, BRADLEY TOYS, INC. PRODUCTION SCHEDULING

TOTAL ITERS	NO. ETAS	ETA REC	ROW IDENT.	CURRENT VALUE	CHOSEN VECTOR	VECTR REMVD	RHS NO.	C/V NO.	CURRENT D/J THETA/PHI	CURRENT RT.
66	66	0	OBJT1	96707.057	Q5-	Q5+	1	2	.49945454	

J(H)	BETA(H)	ROW(I)	PI(I)	B(I)+T*C(I)
0 00000	96707.05799949	OBJT1	1.00000000	.
0 00000	.	OBJT2	.	.
S1	1491.00000000	PRD-1	9.50000000-	.
S2	2106.00000000	PRD-2	9.50000000-	.
S3	2777.00000000	PRD-3	9.50000000-	.
S4	843.00000000	PRD-4	9.50000000-	.
S5	1105.00000000	PRD-5	9.50000000-	.
S6	2932.00000000	PRD-6	9.80600000-	.
S7	1901.00000000	PRD-7	10.11200000-	.
S8	4336.00000000	PRD-8	10.61600000-	.
S9	4578.00000000	PRD-9	10.92200000-	.
S10	1771.00000000	PRD10	9.50000000-	.
S11	4744.00000000	PRD11	10.00400000-	.
S12	3216.00000000	PRD12	9.50000000-	.
X1	1491.00000000	DEM-1	3.11000000	1491.00000000
X2	2106.00000000	DEM-2	3.11000000	2106.00000000
CAP-4	2593.75000000	DEM-3	3.11000000	2777.00000000
X4	843.00000000	DEM-4	3.11000000	843.00000000
X5	1105.00000000	DEM-5	3.11000000	1105.00000000
Q5-	.	DEM-6	2.80400000	2932.00000000
X7	3436.75000000	DEM-7	2.49800000	1901.00000000
Y7	504.75000000	DEM-8	1.99400000	4336.00000000
Q4-	.	DEM-9	1.68800000	4578.00000000
X10	3078.25000000	DEM10	3.11000000	1771.00000000
+ CAP12	220.75000000	DEM11	2.60600000	4744.00000000
Q2-	.	DEM12	3.11000000	3216.00000000
+ CAP-1	1945.75000000	+ CAP-1	.	3436.75000000
+ CAP-2	1330.75000000	+ CAP-2	.	3436.75000000
X3	2777.00000000	+ CAP-3	.	3436.75000000
+ CAP10	358.50000000	+ CAP-4	.	3436.75000000
Q9-	1141.25000000	+ CAP-5	.	3436.75000000
X6	3436.75000000	+ CAP-6	.30600000	3436.75000000
Q8-	899.25000000	+ CAP-7	.61200000	3436.75000000
X8	3436.75000000	+ CAP-8	1.11600000	3436.75000000
X9	3436.75000000	+ CAP-9	1.42200000	3436.75000000
Q11-	1307.25000000	+ CAP10	.	3436.75000000
X11	3436.75000000	+ CAP11	.50400000	3436.75000000
X12	3216.00000000	+ CAP12	.	3436.75000000
Y2	.	CNG-1	.30600000-	.
+ CAP-3	659.75000000	CNG-2	.	.
Q3+	.	CNG-3	.19800000	.
+ CAP-5	2331.75000000	CNG-4	.	.
Y6	.	CNG-5	.19800000	.
Q6+	504.75000000	CNG-6	.19800000	.
Y8	2040.50000000	CNG-7	.19800000	.
Q7+	1535.75000000	CNG-8	.	.
Y9	1141.25000000	CNG-9	.	.
Y11	1307.25000000	CNG10	.19800000	.
Q10+	1307.25000000	CNG11	.	.

Figure E5.3 (*Cont.*)

245800, BRADLEY TOYS, INC. PRODUCTION SCHEDULING

TOTAL ITERS	NO. ETAS	ETA REC	ROW IDENT.	CURRENT VALUE	CHOSEN VECTOR	VECTR REMVD	PHS NO.	C/V NO.	CURRENT D/J THETA/PHI	CURRENT RT.
70	70	0	OBJT1	98496.347	Q8+	Q8-	1	2	1.1534545	

J(H)	BETA(H)	ROW(I)	PI(I)	B(I)+T*C(I)
0 00000	98496.34799954	OBJT1	1.00000000	.
0 00000	.	OBJT2	.	.
S1	1491.00000000	PRD-1	9.50000000-	.
S2	2106.00000000	PRD-2	9.50000000-	.
S3	2777.00000000	PRD-3	9.50000000-	.
S4	843.00000000	PRD-4	9.50000000-	.
S5	1105.00000000	PRD-5	9.50000000-	.
S6	2932.00000000	PRD-6	9.50000000-	.
S7	1901.00000000	PRD-7	9.50000000-	.
S8	4336.00000000	PRD-8	10.00400000-	.
S9	4578.00000000	PRD-9	10.31000000-	.
S10	1771.00000000	PRD10	9.50000000-	.
S11	4744.00000000	PRD11	10.00400000-	.
S12	3216.00000000	PRD12	9.50000000-	.
X1	1491.00000000	DEM-1	3.11000000	1491.00000000
X2	2106.00000000	DEM-2	3.11000000	2106.00000000
+ CAP-4	3493.00000000	DEM-3	3.11000000	2777.00000000
X4	843.00000000	DEM-4	3.11000000	843.00000000
X5	1105.00000000	DEM-5	3.11000000	1105.00000000
Q5-	.	DEM-6	3.11000000	2932.00000000
X7	2143.00000000	DEM-7	3.11000000	1901.00000000
+ CAP-7	2193.00000000	DEM-8	2.60600000	4336.00000000
Q4-	.	DEM-9	2.30000000	4578.00000000
X10	2179.00000000	DEM10	3.11000000	1771.00000000
+ CAP12	1120.00000000	DEM11	2.60600000	4744.00000000
Q2-	.	DEM12	3.11000000	3216.00000000
+ CAP-1	2845.00000000	+ CAP-1	.	4336.00000000
+ CAP-2	2230.00000000	+ CAP-2	.	4336.00000000
X3	2777.00000000	+ CAP-3	.	4336.00000000
+ CAP10	2157.00000000	+ CAP-4	.	4336.00000000
Q9-	242.00000000	+ CAP-5	.	4336.00000000
X6	2932.00000000	+ CAP-6	.	4336.00000000
Q8+	.	+ CAP-7	.	4336.00000000
X8	4336.00000000	+ CAP-8	.50400000	4336.00000000
X9	4336.00000000	+ CAP-9	.81000000	4336.00000000
Q11-	408.00000000	+ CAP10	.	4336.00000000
X11	4336.00000000	+ CAP11	.50400000	4336.00000000
X12	3216.00000000	+ CAP12	.	4336.00000000
Y2	.	CNG-1	.30600000-	.
+ CAP-3	1559.00000000	CNG-2	.	.
Q3+	.	CNG-3	.19800000	.
+ CAP-5	3231.00000000	CNG-4	.	.
+ CAP-6	1404.00000000	CNG-5	.	.
Q6-	.	CNG-6	.	.
Y8	242.00000000	CNG-7	.19800000	.
Q7+	242.00000000	CNG-8	.	.
Y9	242.00000000	CNG-9	.	.
Y11	408.00000000	CNG10	.19800000	.
Q10+	408.00000000	CNG11	.	.

Figure E5.3 (*Cont.*)

245800, BRADLEY TOYS, INC. PRODUCTION SCHEDULING

TOTAL ITERS	NO. ETAS	ETA REC	ROW IDENT.	CURRENT VALUE	CHOSEN VECTOR	VECTR REMVD	RHS NO.	C/V NO.	CURRENT D/J THETA/PHI	CURRENT RT.
73	73	0	OBJT1	98814.335	Q8-	Q8+	1	2	1.3294545	

J(H)	BETA(H)	ROW(I)	PI(I)	B(I)+T*C(I)
0 00000	98814.33599955	OBJT1	1.00000000	.
0 00000	.	OBJT2	.	.
S1	1491.00000000	PRD-1	9.50000000-	.
S2	2106.00000000	PRD-2	9.50000000-	.
S3	2777.00000000	PRD-3	9.50000000-	.
S4	843.00000000	PRD-4	9.50000000-	.
S5	1105.00000000	PRD-5	9.50000000-	.
S6	2932.00000000	PRD-6	9.50000000-	.
S7	1901.00000000	PRD-7	9.50000000-	.
S8	4336.00000000	PRD-8	9.50000000-	.
S9	4578.00000000	PRD-9	10.00400000-	.
S10	1771.00000000	PRD10	9.50000000-	.
S11	4744.00000000	PRD11	10.00400000-	.
S12	3216.00000000	PRD12	9.50000000-	.
X1	1491.00000000	DEM-1	3.11000000	1491.00000000
X2	2106.00000000	DEM-2	3.11000000	2106.00000000
+ CAP-4	3735.00000000	DEM-3	3.11000000	2777.00000000
X4	843.00000000	DEM-4	3.11000000	843.00000000
X5	1105.00000000	DEM-5	3.11000000	1105.00000000
Q5-	.	DEM-6	3.11000000	2932.00000000
X7	1901.00000000	DEM-7	3.11000000	1901.00000000
+ CAP-7	2677.00000000	DEM-8	3.11000000	4336.00000000
Q4-	.	DEM-9	2.60600000	4578.00000000
X10	1937.00000000	DEM10	3.11000000	1771.00000000
+ CAP12	1362.00000000	DEM11	2.60600000	4744.00000000
Q2-	.	DEM12	3.11000000	3216.00000000
+ CAP-1	3087.00000000	+ CAP-1	.	4578.00000000
+ CAP-2	2472.00000000	+ CAP-2	.	4578.00000000
X3	2777.00000000	+ CAP-3	.	4578.00000000
+ CAP10	2641.00000000	+ CAP-4	.	4578.00000000
Q9-	.	+ CAP-5	.	4578.00000000
X6	2932.00000000	+ CAP-6	.	4578.00000000
Q8-	.	+ CAP-7	.	4578.00000000
X8	4336.00000000	+ CAP-8	.	4578.00000000
X9	4578.00000000	+ CAP-9	.50400000	4578.00000000
Q11-	166.00000000	+ CAP10	.	4578.00000000
X11	4578.00000000	+ CAP11	.50400000	4578.00000000
X12	3216.00000000	+ CAP12	.	4578.00000000
Y2	.	CNG-1	.30600000-	.
+ CAP-3	1801.00000000	CNG-2	.	.
Q3+	.	CNG-3	.19800000	.
+ CAP-5	3473.00000000	CNG-4	.	.
+ CAP-6	1646.00000000	CNG-5	.	.
Q6-	.	CNG-6	.	.
+ CAP-8	242.00000000	CNG-7	.	.
Q7-	.	CNG-8	.19800000	.
Y9	.	CNG-9	.	.
Y11	166.00000000	CNG10	.19800000	.
Q10+	166.00000000	CNG11	.	.

Figure E5.3 (*Cont.*)

245800, BRADLEY TOYS, INC. PRODUCTION SCHEDULING

TOTAL ITERS	NO. ETAS	ETA REC	ROW IDENT.	CURRENT VALUE	CHOSEN VECTOR	VECTR REMVD	RHS NO.	C/V NO.	CURRENT D/J THETA/PHI	THETA AT MA
78	78	0	OBJT1	98897.999			1	2	1.4501818	

J(H)	BETA(H)	ROW(I)	PI(I)	B(I)+T*C(I)
0 00000	98897.99999956	OBJT1	1.00000000	.
0 00000	.	OBJT2	.	.
S1	1491.00000000	PRD-1	9.50000000-	.
S2	2106.00000000	PRD-2	9.50000000-	.
S3	2777.00000000	PRD-3	9.50000000-	.
S4	843.00000000	PRD-4	9.50000000-	.
S5	1105.00000000	PRD-5	9.50000000-	.
S6	2932.00000000	PRD-6	9.50000000-	.
S7	1901.00000000	PRD-7	9.50000000-	.
S8	4336.00000000	PRD-8	9.50000000-	.
S9	4578.00000000	PRD-9	9.50000000-	.
S10	1771.00000000	PRD10	9.50000000-	.
S11	4744.00000000	PRD11	9.50000000-	.
S12	3216.00000000	PRD12	9.50000000-	.
X1	1491.00000000	DEM-1	3.11000000	1491.00000000
X2	2106.00000000	DEM-2	3.11000000	2106.00000000
+ CAP-4	3901.00000000	DEM-3	3.11000000	2777.00000000
X4	843.00000000	DEM-4	3.11000000	843.00000000
X5	1105.00000000	DEM-5	3.11000000	1105.00000000
Q5-	.	DEM-6	3.11000000	2932.00000000
X7	1901.00000000	DEM-7	3.11000000	1901.00000000
+ CAP-7	2843.00000000	DEM-8	3.11000000	4336.00000000
Q4-	.	DEM-9	3.11000000	4578.00000000
X10	1771.00000000	DEM10	3.11000000	1771.00000000
+ CAP12	1528.00000000	DEM11	3.11000000	4744.00000000
Q2-	.	DEM12	3.11000000	3216.00000000
+ CAP-1	3253.00000000	+ CAP-1	.	4744.00000000
+ CAP-2	2638.00000000	+ CAP-2	.	4744.00000000
X3	2777.00000000	+ CAP-3	.	4744.00000000
+ CAP10	2973.00000000	+ CAP-4	.	4744.00000000
Q9+	.	+ CAP-5	.	4744.00000000
X6	2932.00000000	+ CAP-6	.	4744.00000000
Q8-	.	+ CAP-7	.	4744.00000000
X8	4336.00000000	+ CAP-8	.	4744.00000000
X9	4578.00000000	+ CAP-9	.	4744.00000000
Q11+	.	+ CAP10	.	4744.00000000
X11	4744.00000000	+ CAP11	.	4744.00000000
X12	3216.00000000	+ CAP12	.	4744.00000000
Y2	.	CNG-1	.30600000-	.
+ CAP-3	1967.00000000	CNG-2	.	.
Q3+	.	CNG-3	.19800000	.
+ CAP-5	3639.00000000	CNG-4	.	.
+ CAP-6	1812.00000000	CNG-5	.	.
Q6-	.	CNG-6	.	.
+ CAP-8	408.00000000	CNG-7	.	.
Q7-	.	CNG-8	.	.
+ CAP-9	166.00000000	CNG-9	.19800000	.
+ CAP11	.	CNG10	.	.
Q10-	.	CNG11	.19800000	.

THETA UNBOUNDED ABOVE THIS VALUE

Figure E5.3 (*Cont.*)

PARAMETRIC COST RANGING

245800, BRADLEY TOYS, INC. PRODUCTION SCHEDULING

TOTAL ITERS	NO. ETAS	ETA REC	ROW IDENT.	CURRENT VALUE	CHOSEN VECTOR	VECTR REMVD	RHS NO.	C/V NO.	CURRENT D/J THETA/PHI	SOLUTION PR
107	107	0	OBJT1	90072.329	Q1+	U12	1	1	.45400000	

J(H)	BETA(H)	ROW(I)	PI(I)	B(I)
0 00000	90072.32999925	OBJT1	1.00000000	.
0 00000	.	OBJT2	.45400000	.
S1	1491.00000000	PRD-1	9.50000000-	.
S2	2106.00000000	PRD-2	9.80600000-	.
S3	2777.00000000	PRD-3	10.31000000-	.
S4	843.00000000	PRD-4	10.41800000-	.
S5	1105.00000000	PRD-5	10.72400000-	.
S6	2932.00000000	PRD-6	11.22800000-	.
S7	1901.00000000	PRD-7	11.33600000-	.
S8	4336.00000000	PRD-8	11.84000000-	.
S9	4578.00000000	PRD-9	12.14600000-	.
S10	1771.00000000	PRD10	12.25400000-	.
S11	4744.00000000	PRD11	12.75800000-	.
S12	3216.00000000	PRD12	13.06400000-	.
X1	1550.00000000	DEM-1	3.11000000	1491.00000000
X2	2750.00000000	DEM-2	2.80400000	2106.00000000
Y10	1481.00000000	DEM-3	2.30000000	2777.00000000
X4	2750.00000000	DEM-4	2.19200000	843.00000000
X5	2750.00000000	DEM-5	1.88600000	1105.00000000
Y6	4228.00000000	DEM-6	1.38200000	2932.00000000
X7	2750.00000000	DEM-7	1.27400000	1901.00000000
Q7+	849.00000000	DEM-8	.77000000	4336.00000000
Y5	2583.00000000	DEM-9	.46400000	4578.00000000
X10	2750.00000000	DEM10	.35600000	1771.00000000
Q10+	979.00000000	DEM11	.14800000-	4744.00000000
Y3	703.00000000	DEM12	.45400000-	3216.00000000
+ CAP-1	1200.00000000	+ CAP-1	.	2750.00000000
Y12	466.00000000	+ CAP-2	.30600000	2750.00000000
X3	2750.00000000	+ CAP-3	.81000000	2750.00000000
Q11-	1994.00000000	+ CAP-4	.91800000	2750.00000000
Y9	3309.00000000	+ CAP-5	1.22400000	2750.00000000
X6	2750.00000000	+ CAP-6	1.72800000	2750.00000000
Q8-	1586.00000000	+ CAP-7	1.83600000	2750.00000000
X8	2750.00000000	+ CAP-8	2.34000000	2750.00000000
X9	2750.00000000	+ CAP-9	2.64600000	2750.00000000
Y11	2460.00000000	+ CAP10	2.75400000	2750.00000000
X11	2750.00000000	+ CAP11	3.25800000	2750.00000000
X12	2750.00000000	+ CAP12	3.56400000	2750.00000000
Y2	59.00000000	CNG-1	.19800000	.
Q3-	27.00000000	CNG-2	.19800000	.
Q2+	644.00000000	CNG-3	.	.
Q9-	1828.00000000	CNG-4	.19800000	.
Q5+	1645.00000000	CNG-5	.19800000	.
Q6-	182.00000000	CNG-6	.	.
Y7	4046.00000000	CNG-7	.19800000	.
Y8	4895.00000000	CNG-8	.	.
Q4+	1907.00000000	CNG-9	.	.
Y4	676.00000000	CNG10	.19800000	.
Q1+	59.00000000	CNG11	.	.

PHI UNBOUNDED ABOVE THIS VALUE

Figure E5.3 (*Concluded*)

Using the computer output supplied, answer the following questions.

a) Draw a graph depicting the following as a function of time, assuming a capacity of 2750 cases per month.

i) Cases demanded
ii) Cases produced
iii) Cases in inventory
iv) Cases of unfilled demand

Explain thoroughly what this graph implies about the optimal operations of Toys, Inc.

b) Give a complete economic interpretation of the dual variables.
c) Give a concise explanation of the righthand-side and cost ranging output.
d) Use the parametric programming of the righthand side as a basis for discussion the optimal production capacity.
e) Use the parametric programming of the cost function as a basis for discussing the ‘‘value" of goodwill loss associated with unfilled demand. (When demand is not met, we lose some goodwill of our customer. What is this loss worth?)

4. *Solving an LP by computer.* Your doctor has found in you a very rare disease, ordinarily incurable, but, in your case, he believes that perhaps something can be done by a series of very elaborate treatments coupled with a strict diet. The treatments are so expensive that it becomes necessary to minimize the cost of the diet.

The diet must provide minimum amounts of the following items: calories, calcium, vitamin A, riboflavin, and ascorbic acid. Your daily requirement for these items (in the above order) may be determined by reading off the numerical values corresponding to the first five letters of your name on Table 5.4. The following are the units used: 10^2 calories, 10^{-2} grams, 10^2 international units, 10^{-1} milligrams, and milligrams.

Table 5.4 Diet Requirements

	Diet	*Product* X		*Diet*	*Product* X
A	7	63	N	6	91
B	60	52	O	10	45
C	83	59	P	32	82
D	10	85	Q	51	98
E	39	82	R	47	67
F	59	58	S	20	97
G	38	50	T	66	28
H	30	69	U	78	54
I	65	44	V	81	33
J	27	26	W	81	59
K	91	30	X	61	61
L	68	43	Y	0	39
M	49	90	Z	86	83

Your choice of foods is somewhat limited because you find it financially advantageous to trade at a discount store that has little imagination. You can buy: (1) wheat flour (enriched), (2) evaporated milk, (3) cheddar cheese, (4) beef liver, (5) cabbage, (6) spinach, (7) sweet potatoes, (8) lima beans (dried).

The nutritional values per dollar spent have been tabulated by Dantzig (who regularly patronizes the store) in Table 5.5. In addition, the store features a grayish powder, Product X, sold in bulk, whose nutritional values per unit cost are also given in Table 5.4. The units (same order of items as before) are 10^3 calories/dollar, 10^{-1} grams/dollar, 10^3 international units/dollar, 10^{-1} milligrams/dollar, milligrams/dollar.

Your doctor has coded your diet requirements and the nutritional properties of Product X under the first five letters of your name in Table 5.4.

a) Find your minimum-cost diet and its cost.
b) How much would you be willing to pay for pure vitamin A? pure riboflavin?

Table 5.5 Nutritive Values of Common Foods Per Dollar of Expenditure*

Commodity	*Calories* (1000)	*Protein* (grams)	*Calcium* (grams)	*Iron* (mg.)	*Vitamin A* (1000 I.U.)	*Thiamine* (mg.)	*Riboflavin* (mg.)	*Niacin* (mg.)	*Ascorbic Acid* (mg.)
1. Wheat flour (enriched)	44:7	1411	2:0	365	—	55.4	33.3	441	—
5. Corn meal	36.0	897	1.7	99	30.9	17.4	7.9	106	—
15. Evaporated milk (can)	8.4	422	15.1	9	26.0	3.0	23.5	11	60
17. Oleomargarine	20.6	17	.6	6	55.8	.2	—	—	—
19. Cheese (cheddar)	7.4	448	16.4	19	28.1	.8	10.3	4	—
21. Peanut butter	15.7	661	1.0	48	—	9.6	8.1	471	—
24. Lard	41.7	—	—	—	.2	—	.5	5	—
30. Liver (beef)	2.2	333	.2	139	169.2	6.4	50.8	316	525
34. Pork loin roast	4.4	249	.3	37	—	18.2	3.6	79	—
40. Salmon, pink (can)	5.8	705	6.8	45	3.5	1.0	4.9	209	—
45. Green beans	2.4	138	3.7	80	69.0	4.3	5.8	37	862
46. Cabbage	2.6	125	4.0	36	7.2	9.0	4.5	26	5369
50. Onions	5.8	166	3.8	59	16.6	4.7	5.9	21	1184
51. Potatoes	14.3	336	1.8	118	6.7	29.4	7.1	198	2522
52. Spinach	1.1	106	—	138	918.4	5.7	13.8	33	2755
53. Sweet potatoes	9.6	138	2.7	54	290.7	8.4	5.4	83	1912
64. Peaches, dried	8.5	87	1.7	173	86.8	1.2	4.3	55	57
65. Prunes, dried	12.8	99	2.5	154	85.7	3.9	4.3	65	257
68. Lima beans, dried	17.4	1055	3.7	459	5.1	26.9	38.2	93	—
69. Navy beans, dried	26.9	1691	11.4	792	—	38.4	24.6	217	—

* *Source*: G. B. Dantzig, *Linear Programming and Extension*, Princeton University Press, Princeton, N.J., 1963.

c) A new food has come out (called GLUNK) having nutritional values per dollar of 83, 17, 25, 93, 07 (values are in the same order and same units as for Product X). Would you want to include the new food in your diet?
d) By how much would the cost of lima beans have to change before it would enter (or leave, as the case may be) your diet?
e) Over what range of values of cost of beef liver would your diet contain this wonderful food?
f) Suppose the cost of foods 1, 3, 5, 7, and 9 went up 10% but you continued on the diet found in (a). How much would you be willing to pay for pure vitamin A? pure riboflavin?
g) If the wheat flour were enriched by 10 units of vitamin A without additional cost, would this change your diet?

ACKNOWLEDGMENTS

Exercise 1 is based on Chapter 14 of *Applied Linear Programming*, by Norman J. Driebeck, Addison-Wesley Publishing Company, Inc., 1969.

Exercises 2 and 3 are based on cases with the same names written by one of the authors.

Exercise 4 is a variation of the diet problem used by John D.C. Little of the Massachusetts Institute of Technology.